Given \( f(x) = a_0 + a_1(x-c) + a_2(x-c)^2 + \ldots = \sum_{n=0}^{\infty} a_n(x-c)^n \) is differentiable (and therefore continuous) on the interval \((c - R, c + R)\) then the following is true:

\[
\begin{align*}
  f^{(0)}(x) &= a_0 + a_1(x-c) + a_2(x-c)^2 + a_3(x-c)^3 + a_4(x-c)^4 + \cdots \\
  f^{(1)}(x) &= a_1 + 2a_2(x-c) + 3a_3(x-c)^2 + 4a_4(x-c)^3 + \cdots \\
  f^{(2)}(x) &= 2a_2 + 3!a_3(x-c) + 4\cdot 3a_4(x-c)^2 + \cdots \\
  f^{(3)}(x) &= 3!a_3 + 4!a_4(x-c) + \cdots \\
  &\quad \vdots \\
  f^{(n)}(x) &= n!a_n + (n+1)!a_{n+1}(x-c) + (n+2)!a_{n+2}(x-c)^2 + \cdots
\end{align*}
\]

Evaluating each of these derivatives at \( x = c \) yields

\[
\begin{align*}
  f^{(0)}(c) &= 0!a_0 \\
  f^{(1)}(c) &= 1!a_1 \\
  f^{(2)}(c) &= 2!a_2 \\
  f^{(3)}(c) &= 3!a_3 \\
  \vdots \\
  f^{(n)}(c) &= n!a_n
\end{align*}
\]
and, in general, \( f^{(n)}(c) = n!a_n \). By solving for \( a_n \), you can find the coefficients of the power series representation of \( f(x) \) are:

\[
a_n = \frac{f^{(n)}(c)}{n!}. \]

Definitions of nth Taylor Polynomial and nth Maclaurin Polynomial

If \( f \) has \( n \) derivatives at \( c \), then the polynomial

\[
P_n(x) = f(c) + f'(c)(x-c) + \frac{f''(c)}{2!}(x-c)^2 + \frac{f'''(c)}{3!}(x-c)^3 + \cdots + \frac{f^{(n)}(c)}{n!}(x-c)^n
\]
is called the \textbf{nth Taylor polynomial for} \( f \) \textbf{at} \( c \). If \( c = 0 \), then

\[
P_n(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \cdots + \frac{f^{(n)}(0)}{n!}x^n
\]
is called the \textbf{nth Maclaurin polynomial for} \( f \).
Derivation of series for $e^x$, $\cos x$, $\sin x$, and $\ln x$

\[
f(x) = e^x \quad f(x) = \cos(x) \quad \text{and} \quad f(x) = \sin(x) \quad f(x) = \ln(x)
\]
Maclaurin Series for a Composite Function:

Find the Maclaurin series.
1. \( f(x) = \sin x^2 \) 
2. \( f(x) = \cos \sqrt{x} \) 
3. \( f(x) = e^{-3x} \) 
4. \( f(x) = \ln(1 + x^2) \)

Find nth degree Taylor or Maclaurin series

1. \( f(x) = e^{-x}, \quad n = 5, \quad c = 0 \) 
2. \( f(x) = \cos \pi x, \quad n = 4, \quad c = 0 \)
3. \( f(x) = \sqrt[3]{x}, \quad n = 3, \quad c = 8 \)