Polar Graphs

How to plot polar coordinates: \((r, \theta)\)

The origin is now called the pole. \(r\) = a directed distance from O to P.

\(\theta\) = directed angle, counterclockwise from polar axis to \(\overline{OP}\)

Special Polar Graphs

Circles: \(r = a\)

- \(r = 3\)
- \(r = \pm 2a \cos \theta\)
- \(r = -2 \cos \theta\)
- \(r = \pm 2a \sin \theta\)
- \(r = 6 \sin \theta\)

Use Graphing Calculator

For TI-83: MODE is Pol, Deg. or Rad,

WINDOW must correspond with Deg. or Rad.

Graph the following with calculator: \(r = 3 + 3 \cos \theta\), \(r = 3 + 4 \sin \theta\), \(r = 5 - 3 \sin \theta\)
**Limaçons**  $r = a \pm b \cos \theta$  $r = a \pm b \sin \theta$

$r = 1 + 3 \cos \theta$  \hspace{1cm}  $r = 2 - 2 \sin \theta$  \hspace{1cm}  $r = 3 + 2 \cos \theta$  \hspace{1cm}  $r = 4 + 2 \sin \theta$

Limacon: [https://youtu.be/MT1j6vdX2aI](https://youtu.be/MT1j6vdX2aI)

**Lemniscate (a figure 8):**  $r^2 = a^2 \cos(2\theta)$  \hspace{1cm}  $r^2 = a^2 \sin(2\theta)$

$r^2 = 25 \cos(2\theta)$  \hspace{1cm}  $r^2 = 16 \sin(2\theta)$

Lemniscate: [https://youtu.be/tvPsjmcN2Yg](https://youtu.be/tvPsjmcN2Yg)

**Rose Curves:**  $r = a \cos(n\theta)$  \hspace{1cm}  $r = a \sin(n\theta)$

$r = 3 \cos(3\theta)$  \hspace{1cm}  $r = 3 \cos(2\theta)$  \hspace{1cm}  $r = 3 \sin(5\theta)$  \hspace{1cm}  $r = 3 \sin(2\theta)$

Rose curve: [https://youtu.be/tvPsjmcN2Yg](https://youtu.be/tvPsjmcN2Yg)
Slope and Tangent Lines

Given that $x = r \cos \theta$, $y = r \sin \theta$ and $r = f(\theta)$, find $\frac{dy}{dx}$.

1. Find $\frac{dy}{dx}$ and the slopes of the tangent lines at the given values of $\theta$.

   $r = -1 + \sin \theta$, $\theta = 0$ and $\theta = \pi$
2. Determine the equation of the tangent line to \( r = 3 + 8\sin \theta \) at \( \theta = \frac{\pi}{6} \)

**Tangent Lines**

There is a horizontal tangent line if \( \frac{dy}{d\theta} = 0 \) and \( \frac{dx}{d\theta} \neq 0 \)

There is a vertical tangent line if \( \frac{dx}{d\theta} = 0 \) and \( \frac{dy}{d\theta} \neq 0 \)
3. Find all the points of vertical and horizontal tangency:  \( r = -1 + \sin \theta \)

https://www.youtube.com/watch?v=4SpQ9iOxd_E

4. Review quadratic formula. Solve:  \( 4 \cos^2 \theta + \cos \theta - 2 = 0 \) within interval \([0, 2\pi)\)
**Tangents at the Pole**

There is a tangent line at the pole if \( f(\theta) = 0 \) and \( f'(\theta) \neq 0 \), the line is: \( \theta = \alpha \)

4. Find all the tangents at the pole: \( r = 2\cos 3\theta \)
10.4 P. 722:59, 61, 63, 67, 69, 71, 73 AND

A. Find all the points of vertical and horizontal tangency: \( r = 1 + \cos \theta \)

B. Determine the equation of the tangent line to \( r = \sin(4\theta)\cos \theta \) at \( \theta = \frac{\pi}{6} \)

C. Determine the equation of the tangent line to \( r = \theta \sin(3\theta) \) at \( \theta = \frac{\pi}{2} \)

D. Determine the equation of the tangent line to \( r = 1 + 2\cos \theta \) at \( \theta = \frac{\pi}{4} \)

E. Find all the points of vertical and horizontal tangency: \( r = 2 + 4\cos \theta \)
   (you will need to use the quadratic formula)

Answers:

1. Horizontal: \( (r, \theta) = \left( \frac{3}{2}, \frac{\pi}{3} \right), \left( \frac{3}{2}, \frac{5\pi}{3} \right) \)

2. \( y = \frac{1}{3\sqrt{3}} x + \frac{1}{4} \)

3. \( y = -\frac{2}{\pi} x - \frac{\pi}{2} \)

4. \( y = \frac{1}{7} (1 - 2\sqrt{2}) \left( x - \frac{1 + \sqrt{2}}{\sqrt{2}} \right) + \frac{1 + \sqrt{2}}{\sqrt{2}} \)

5. Bonus