Solve the following with the given interval. Use exact values where possible.

1. \(10 \sin^2 \theta + 3 \sin \theta - 1 = 0\), \([0, 2\pi]\)
2. \(3 \tan \theta \sin \theta + \tan \theta = 0\), \([0, 2\pi]\)
3. \(8 \cos^2 x - \cos x - 1 = 0\), \([0, 2\pi]\)
4. \(6 \cos^2 \theta + \cos \theta - 2 = 0\), \([0, 2\pi]\)
5. \(5 \tan^2 x - 2 \tan x - 1 = 0\), \([0, \pi]\)
6. \(\frac{1}{2} - \pi \cos(2x) = 0\), \([-\pi/2, \pi/2]\)
7. \(5 - \pi \csc^2 \pi x = 0\); \([0, 2]\)
8. \(2 \sec^2 x - \tan x = 4\), \([0, 2\pi]\)

9. Find the vertex, focus, and directrix of the parabola: \(x^2 - 2x = 8y - 1\)

10. Find the center, foci, vertices, and graph the equation.
\[
\frac{(x-3)^2}{49} + \frac{(y+1)^2}{4} = 1
\]

11. Complete the square and write in standard form.

(a) \(25x^2 + 4y^2 + 100x - 32y + 64 = 0\) \hspace{1cm} (b) \(18y^2 - 5x^2 + 30x + 72y - 63 = 0\)

12. Find the equations of the asymptotes of a hyperbola AND graph the equation.

(a) \(\frac{(y+3)^2}{4} - \frac{(x-2)^2}{9} = 1\) \hspace{1cm} (b) \(\frac{(x+1)^2}{16} - \frac{(y-2)^2}{9} = 1\)

13. Find the equation of the parabola described: Vertex \((-3, 2)\); Focus \((-5, 2)\)

14. Find the equation of the ellipse with center \((-3, 1)\), focus \((-3, 0)\) and vertex \((-3, 3)\).

15. Find the equation of the hyperbola with focus \((-2, 4)\); center \((1, 4)\) and vertex \((0, 4)\).

16. Jamie and Jason want to exchange secrets from across a crowded whispering gallery. Recall that a whispering gallery is a room which, in cross section, is half of an ellipse. If the room is 40 feet high at the center and 100 feet wide at the floor, how far from the outer wall should each of them stand so that they will be positioned at the foci of the ellipse?
17. The mirror in Carl's flashlight is a paraboloid of revolution. If the mirror is 12 centimeters in diameter and 4 centimeters deep, where should the light bulb be placed so it is at the focus of the mirror?

18. Determine the appropriate rotation formulas (with EXACT values) to use so that the new equation contains no $xy$-term. NO DECIMALS

$$x^2 + \sqrt{3}xy + 2y^2 - 10 = 0$$

19. Graph the following transformed equation, if the angle of rotation, $\theta = 60^\circ$

$$\frac{x^2}{4} + \frac{y^2}{16} = 1$$

20. Graph the following transformed equation, if the angle of rotation, $\theta = 45^\circ$

$$\frac{x^2}{25} - \frac{y^2}{16} = 1$$

Provided Formulas:

$$\cot 2\theta = \frac{A - C}{B}$$

$$x = x' \cos \theta - y' \sin \theta$$

$$y = x' \sin \theta + y' \cos \theta$$

$$\sin \theta = \sqrt{\frac{1 - \cos 2\theta}{2}}$$

$$\cos \theta = \sqrt{\frac{1 + \cos 2\theta}{2}}$$

$$\sin 2\theta = 2 \sin \theta \cos \theta$$

$$\cos 2\theta = \cos^2 \theta - \sin^2 \theta = 2 \cos^2 \theta - 1 = 1 - 2 \sin^2 \theta$$
1. \(10 \sin^2 \theta + 3 \sin \theta - 1 = 0 \) \(0, 2\pi\)
   
   \((5 \sin \theta - 1)(2 \sin \theta + 1) = 0\)
   
   \(\sin \theta = \frac{1}{5}, \sin \theta = -\frac{1}{2}\)
   
   \(\theta = 0.201, 2.94\) \(\theta = \frac{7\pi}{6}, \frac{11\pi}{6}\)

2. \(3 \tan \theta \sin \theta + \tan \theta = 0\)
   
   \(\tan \theta (3 \sin \theta + 1) = 0\)
   
   \(\tan \theta = 0, \sin \theta = -\frac{1}{3}\)
   
   \(\theta = 0, \pi\) \(\theta = 3.48, 5.94\)

3. \(8 \cos^2 x - \cos x - 1 = 0\)
   
   \(\cos x = \frac{1 \pm \sqrt{1 - 4(8)(-1)}}{16}\)
   
   \(\cos x = \frac{1 \pm \sqrt{33}}{16}\)
   
   \(\cos x = 0.422, -0.296\)
   
   \(x = 1.135, 5.148\) \(x = 4.412\)

4. \(\sin^2 \theta + \cos \theta - 2 = 0\)
   
   \((3 \cos \theta + 2)(2 \cos \theta - 1) = 0\)
   
   \(\cos \theta = -\frac{2}{3}, \cos \theta = \frac{1}{2}\)
   
   \(\theta = 2.301, \frac{\pi}{3}, \frac{5\pi}{3}\)
   
   \(3.983\)

5. \(\tan^2 x - 2 \tan x - 1 = 0\) \([0, \pi]\)
   
   \(\tan x = \frac{2 \pm \sqrt{4 - 4(5)(-1)}}{10}\)
   
   \(\tan x = \frac{2 \pm \sqrt{124}}{10}\)
   
   \(\tan x = 0.689, -0.289\)
   
   \(x = 0.603\) \(x = 2.859\)

6. \(\frac{1}{2} - \pi \cos(2x) = 0\) \([-\frac{\pi}{2}, \frac{\pi}{2}]\)
   
   \(-\pi \cos(2x) = -\frac{1}{2}\)
   
   \(\cos(2x) = \frac{1}{2}\)
   
   \(2x = \cos^{-1}\left(\frac{1}{2}\right)\) \(2x: [-\pi, \pi]\)
   
   \(2x = 1.41, -1.41\)
   
   \(x = 0.705, -0.705\)

7. \(5 - \pi \csc^2(\pi x) = 0\) \([0, 2]\)
   
   \(-\pi \csc^2(\pi x) = -5\)
   
   \(\csc^2(\pi x) = \frac{5}{\pi}\)
   
   \(\sin^2(\pi x) = \frac{\pi}{5}\)
   
   \(\sin(\pi x) = \pm \sqrt{\frac{\pi}{5}}\)
   
   \(\pi x = \sin^{-1}\left(\sqrt{\frac{\pi}{5}}\right), \pi x = \sin^{-1}\left(-\sqrt{\frac{\pi}{5}}\right)\) \(\pi x: [0, 2\pi]\)
   
   \(\pi x = 0.915, 2.226\) \(\pi x = 5.368, 4.057\)
   
   \(x = 0.291, 0.709\) \(x = 1.709, 1.291\)
8. \(2 \sec^2 x - \tan x = 4\) \([0, 2\pi]\)
\(2(1 + \tan^2 x) - \tan x - 4 = 0\)
\(2 + 2\tan^2 x - \tan x - 4 = 0\)
\(2\tan^2 x - \tan x - 2 = 0\)
\(\tan x = \frac{1 \pm \sqrt{1 - 4(2)(-2)}}{4} = \frac{1 \pm \sqrt{17}}{4}\)
\(\tan x = 1.28\) \(\tan x = -0.7808\)
\(x = 0.908, 4.049\) \(x = 5.62, 2.479\)

9. \(x^2 - 2x = 8y - 1\)
\(x^2 - 2x + (-1)^2 = 8y - 1 + \frac{(-1)^2}{4}\)
\((x-1)^2 = 8y\)

Vertex \((1, 0)\)
Focus \((1, 2)\)
Directrix: \(y = -2\)

10. \(\frac{(x-3)^2}{49} + \frac{(y+1)^2}{4} = 1\)

\(C (3, -1)\)

Vertices \((10, -1), (-4, -1)\)

\(c^2 = 49 - 4\)
\(c^2 = 45\)
\(c = \pm 3\sqrt{5}\)

Foci \((3 \pm 3\sqrt{5}, -1)\)
11. (a) \[ 25x^2 + 100x + 4y^2 - 32y = -164 \]
\[ 25(x^2 + 4x + \left(\frac{4}{25}\right)^2) + 4(y^2 - 8y + \left(\frac{4}{4}\right)^2) = -164 + 100 + 64 \]
\[ 25(x-2)^2 + 4(y-4)^2 = \frac{100}{100} \]
\[ \frac{(x-2)^2}{4} + \frac{(y-4)^2}{25} = 1 \]

(b) \[ 18y^2 + 72y - 5x^2 + 30x = 63 \]
\[ 18(y^2 + 4y + \left(\frac{2}{18}\right)^2) - 5(x^2 - 6x + \left(\frac{3}{18}\right)^2) = 63 + 72 - 45 \]
\[ 18(y+2)^2 - 5(x-3)^2 = \frac{90}{90} \]
\[ \frac{(y+2)^2}{5} - \frac{(x-3)^2}{18} = 1 \]

12. (a) \[ \frac{(y+3)^2}{4} - \frac{(x-3)^2}{9} = 1 \]
\[ C(2, -3) \]
Asymptotes: \( (y+3) = \pm \frac{2}{3} (x-2) \)

(b) \[ \frac{(x+1)^2}{16} - \frac{(y-2)^2}{9} = 1 \]
Asymptotes: \( (y-2) = \pm \frac{3}{4} (x+1) \)
13. \( V(-3, 2) \) \( F(-5, 2) \)
\[ (y-2)^2 = -8(x+3) \]

14. \( C(-3, 1) \) \( F(-3, 0) \) \( V(-3, 3) \)
\[ \frac{(x+3)^2}{3} + \frac{(y-1)^2}{4} = 1 \]

15. \( E(-2, 4) \) \( C(1, 4) \) \( V(0, 4) \) Hyperbola
\[ \frac{(x-1)^2}{1} - \frac{(y-4)^2}{8} = 1 \]

16. \( C^2 = a^2 - b^2 \)
\( C^2 = 50^2 - 40^2 \)
\( C^2 = 2500 - 1600 \)
\( C^2 = 900 \)
\( C = 30 \)

17. 3D from outer walls

18. 12 \( (6, 4) \) parabola
\[ (x-6)^2 = 4a(y-k) \]
\[ (6-0)^2 = 4a(4-0) \]
\( 36 = 4a(4) \)
\[ a = \frac{9}{4} \]
18. \( x^2 + 13xy + 2y^2 - 10 = 0 \)

\[
\cot 2\theta = \frac{1 - 2}{13} = \frac{-1}{13}
\]

\[\theta = 60^\circ\]

\[2\theta = 120^\circ\]

\[\theta = 60^\circ\]

\[
\begin{align*}
x &= \hat{x} \cos 60^\circ - \hat{y} \sin 60^\circ \\
y &= \hat{x} \sin 60^\circ + \hat{y} \cos 60^\circ
\end{align*}
\]

\[
\begin{align*}
x &= \frac{1}{3} \hat{x} - \frac{\sqrt{3}}{2} \hat{y} \\
y &= \frac{\sqrt{3}}{2} \hat{x} + \frac{1}{2} \hat{y}
\end{align*}
\]

19.

20.