Graphing a Polynomial Function

End Behavior of a Graph.

The end behavior is what happens to the graph as \( x \) approaches positive and negative infinity. In other words, as \( x \) takes on very large values, what happens to the graph? And as \( x \) takes on very small values, what happens to the graph.

In a polynomial function, the leading term will determine the end behavior of the graph. We will substitute very large values for \( x \) into the leading term and see if the answer is positive or negative. If the answer is positive, the graph will go up on the right hand side. If the answer is negative, the graph will go down on the right hand side. We will then substitute very small (negative) values for \( x \) into the leading term and see if the answer is positive or negative. If the answer is positive, the graph will go up on the left hand side. If the answer is negative, the graph will go down on the left hand side.

Zeros or Roots of the Graph:

These are the \( x \)-intercepts of the graph. The way the graph crosses the \( x \)-axis is determined by the multiplicity of the roots.

**Multiplicity of One:** The graph will just pass through the \( x \)-axis.

**Even Multiplicity:** The graph will “bounce off” the axis. Like a parabola.

**Odd Multiplicity:** The graph will pass through the axis, but will look like the cubic function. I call this a “squiggle”

Procedure of Graphing a Polynomial Function:

1. Determine the end behavior of the graph.
2. Determine all zeros and their multiplicity. Sometimes you will have to factor to get these. If not factorable, then we will use the rational root theorem.
3. Find the \( y \)-intercept.
4. Plot all these points and play “connect the dots.”
4.2 Graphing Polynomials

① Determine end behavior
② Find the y-intercept (Let x = 0)
③ Find the x-intercepts AND determine their multiplicity.
④ Sketch graph.

End Behavior:

Leading term determines end behavior
(biggest exp)

If leading term has an even power,
then

If leading term has an odd power,
then

\[ f(v) = -2x^5 - 3x^2 + 4 \]
lead

term

\[ f(v) = (3x-1)^2(x+2)^2 \]

\[ 9x^2 \cdot x^2 = 9x^4 \implies (up \ on \ both \ ends) \]
**y-intercept:**

\[ f(x) = 4x^4 - 3x^2 - 1 \]

Let \( x = 0 \)

\[ y = 4(0)^4 - 3(0)^2 - 1 \]

\[ y = -1 \]

\[ f(x) = 0(x + 2)(x - 1)(2x - 3) \]

Let \( x = 0 \)

\[ y = -(0 + 2)(0 - 1)(0 - 3) \]

\[ y = -6 \]

**x-intercepts AND multiplicity**

\[ f(x) = (x - 2)(x - 3)(x - 5) \]

\[ 0 = (x - 2)(x - 3)(x - 5) \]

\[ x - 2 = 0 \quad x - 3 = 0 \quad x - 5 = 0 \]

\[ x = 2 \quad x = 3 \quad x = 5 \]

Let \( y = 0 \) (factor)?

Set each factor = 0 and solve.

\[ f(x) = -\frac{1}{2}x(x + 3)(x - 1)^2 \]

\[ 0 = -\frac{1}{2}x(x + 3)(x - 1)^2 \]

\[ -\frac{1}{2}x = 0 \quad x + 3 = 0 \quad x - 1 = 0 \]

\[ x = 0 \quad x = -3 \quad x = 1 \]

Double root bounces off x-axis
\[ f(x) = (x - 1)^2 (x + 2)^3 (x - 4) \]

\[ 0 = (x - 1)^2 (x + 2)^3 (x - 4) \]

- \[ x - 1 = 0 \] \quad \text{double root or zero} \quad x = 1
- \[ x + 2 = 0 \] \quad \text{triple root} \quad x = -2
- \[ x - 4 = 0 \] \quad \text{single root} \quad x = 4

\text{Double root "bounces" off x-axis}
\text{Triple root "squiggles" thru x-axis}
\text{Single root just passes thru}

34. \[ f(x) = 2x^3 - 6x \]

\[ 0 = 2x(x^2 - 3) \]

- \[ 2x = 0 \] \quad \text{GCF}
- \[ x = 0 \]

- \[ x^2 - 3 = 0 \]
- \[ x = \pm \sqrt{3} \]

3 x-intercepts:
\[ 0, \sqrt{3}, -\sqrt{3} \]

36. \[ f(x) = x^3 + 3x^2 - x - 3 \]

\[ 0 = x^3 + 3x^2 - x - 3 \]

\[ 0 = x^2(x + 3) - 1(x + 3) \]

\[ 0 = (x + 3)(x^2 - 1) \]

\[ 0 = (x + 3)(x + 1)(x - 1) \]

\[ x = -3 \quad x = -1 \quad x = 1 \]

\text{All single roots}
44. \( f(x) = 4x^4 - 3x^2 - 1 \) factor

\[ 0 = (4x^2 + 1)(x^2 - 1) \]

\[ 0 = (4x^2 + 1)(x + 1)/(x - 1) \]

\[ 4x^2 + 1 = 0 \]

\[ x = -\frac{1}{\sqrt{4}} \]

\[ x = -\frac{1}{2} \] (imaginary)

No \( x \)-intercepts.

Graph:

\[ f(x) = -3(5x + 1)(x - 2)^2 \]

Degree of leading term: \(-3 \cdot 25x^2 \cdot x^2 = -75x^4\)

Down on both ends

2) \( y \)-intercept:

\[ y = -3(0 + 1)(0 - 2)^2 \]

\[ y = -3(1)(4) \]

\[ y = -12 \]

3) \( x \)-intercepts:

\[ 0 = -3(5x + 1)(x - 2)^2 \]

\[ 5x + 1 = 0 \]

\[ x = -\frac{1}{5} \]

\[ x - 2 = 0 \]

\[ x = 2 \] Both double
\[ f(x) = -3x^4 + 3x^3 \]

1. **End:** \(-3x^4\) \text{ down on both}
2. **y:** \(y = 0^4 + 0^3 = 0\)
3. **x:** \(0 = -3x^3(x - 1)\)
   - \(-3x^3 = 0\) \(x - 1 = 0\) \text{ single}
   - \(x = 1\)
   - \(x = 0\) \text{ triple}
\[ f(x) = \frac{1}{(x+2)^2} \cdot (x+1) \cdot (x-3)^3 \]

1) \( \text{End: } x^2 \cdot x' \cdot x^3 = x^6 \)

2) \( y = (0+2)^2 \cdot (0+1) \cdot (0-3)^3 \)
   \[ y = (4 \cdot 1) \cdot (-27) = -108 \]

3) \( x \):
   \[ x+2 = 0 \quad x+1 = 0 \quad x-3 = 0 \]
   \[ x = -2 \quad x = -1 \quad x = 3 \]
   \[ \text{double} \quad \text{single} \quad \text{triple} \]

(4.2 p.285: 29-53 odd)