INTERMEDIATE ALGEBRA

Math 0310
RELATIONS AND FUNCTIONS

Cartesian Coordinate System, Domain and Range, Function Notation, Lines, Linear Inequalities
We use a rectangular coordinate system to help us “map” out relations. The coordinate grid has a horizontal axis and a vertical axis. Where these two axes intersect is called the origin. The grid is also divided into 4 quadrants. Traditionally, we use Roman numerals to label these 4 quadrants.

Let’s begin by plotting some ordered pairs.

A=(1, 4)   B=(0,2)  
C=(-3,5)   D=(-4,0)  
E=(-2,-4)  F=(0,-3)  
G=(3,-1)  

Find the ordered pair associated with the given points.
We also use the coordinate system to graph solutions of equations in two variables.

One of the most common equations we graph is linear equations. **LINES** There are several methods to graph lines: plot points by creating a table of values, plot the intercepts, use the slope and y-intercept.

How do you know if the graph of an equation is a line?

\[ y + 2x^2 = 5 \]  \[ 2y + 4x = 11 \]  \[ y = \sqrt{2x + 1} \]
\[ x - 9 = 0 \]  \[ y = 5 \]  \[ y^2 + 2x^2 = 5 \]

Make a Table of Values to graph the following:

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>y</td>
</tr>
<tr>
<td>5</td>
<td>20</td>
</tr>
</tbody>
</table>

What happens when we do NOT have a linear equation?

What happens when \( x \) is squared and \( y \) is not?
Make a Table of Values to graph the following:

\[ y = x^2 + 2 \]

\[ y = 3 - x^2 \]

What happens when \( y \) is squared and \( x \) is not?

Make a Table of Values to graph the following:

\[ x = y^2 + 3 \]

\[ x = 1 - y^2 \]
What happens when $x$ is under a square root sign?

Make a Table of Values to graph the following:

\[ y = \sqrt{x + 3} \quad \text{and} \quad y = \sqrt{x - 2} \]
Cartesian Coordinate System

1. Find the ordered pairs associated with the given points:

Make a table of values and graph the following equations on graph paper provided. (next page)

2. \( y = 2x - 3 \)

3. \( x + 2y = 4 \)

4. \( \frac{1}{2}x + 2 = y \)

5. \( 2x + 3y = 0 \)

6. \( y = x^2 - 5 \)

7. \( y = 9 - x^2 \)

8. \( x = 1 - y^2 \)

9. \( y = \sqrt{x-4} \)

10. \( y = \sqrt{x+2} \)
<table>
<thead>
<tr>
<th>Equation</th>
<th>Graph</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y = 2x - 3$</td>
<td><img src="image1" alt="Graph" /></td>
</tr>
<tr>
<td>$x + 2y = 4$</td>
<td><img src="image2" alt="Graph" /></td>
</tr>
<tr>
<td>$\frac{1}{2}x + 2 = y$</td>
<td><img src="image3" alt="Graph" /></td>
</tr>
<tr>
<td>$2x + 3y = 0$</td>
<td><img src="image4" alt="Graph" /></td>
</tr>
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<td>$y = x^2 - 5$</td>
<td><img src="image5" alt="Graph" /></td>
</tr>
<tr>
<td>$y = 9 - x^2$</td>
<td><img src="image6" alt="Graph" /></td>
</tr>
<tr>
<td>$x = 1 - y^2$</td>
<td><img src="image7" alt="Graph" /></td>
</tr>
<tr>
<td>$y = \sqrt{x - 4}$</td>
<td><img src="image8" alt="Graph" /></td>
</tr>
<tr>
<td>$y = \sqrt{x + 2}$</td>
<td><img src="image9" alt="Graph" /></td>
</tr>
</tbody>
</table>
A relation is a set of ordered pairs. The domain is the set of all first coordinates and the range is the set of all second coordinates.

\{(Bob, tie), (Carly, bow), (Juan, flower), (Bob, scarf)\}

The domain would be \{Bob, Carly, Juan\}

The range would be \{tie, bow, flower, scarf\}

A function is a relation such that no two ordered pairs have the same first coordinate.

An example of a function is:
\{(Sally, pink), (Carly, blue), (Adam, green), (Bob, blue)\}

The domain would be \{Sally, Carly, Adam, Bob\}

The range would be \{pink, blue, green\}

Another example of a function is:
\[A = \{(x, y) : y = 3x - 5, x = -2, 0, 3\}\]
\[B = \{(x, y) : y = \sqrt{x + 10}, x = -6, -1, 6\}\]

Find the domain and range.
What if we have an infinite number of ordered pairs?
We cannot make a list, but we can draw a picture of the relation.

What is the domain?

What is the range?

How can we determine if a graph is a function?
Remember definition: no two ordered pairs have the same first coordinate.
This leads to the **vertical line test**.

Do the following graphs represent a function? Find the domain and range.
Relations and Functions

Do all work on notebook paper. All work should be neat and organized.

Write the following sets as sets of ordered pairs and identify the domain and range.

1. \[ V = \left\{ (x, y) : y = -2x + 7, x = -5, 0, 2, 1, \frac{3}{2} \right\} \]

2. \[ Z = \left\{ (x, y) : y = |x| + 3, x = -2, 0, 2 \right\} \]

3. \[ X = \left\{ (x, y) : y = \sqrt{2x-1}, x = 1, 3, 5, \frac{1}{2} \right\} \]

Determine the domain and range of each relation whose graph is given. Determine which of the following are graphs of functions.

4.

5.

6.

7.
In algebra, we use function notation:

Non-function notation: \( y = x^2 \)
Re-written with function notation: \( f(x) = x^2 \)
   We read this as “\( f \) of \( x \)”
\[
\begin{align*}
   f(3) &= (3)^2 \\
   f(3) &= 9 \\
   f(*) &= *^2 \\
   f(\Theta) &= \Theta^2 \\
   f(\forall) &= \forall^2
\end{align*}
\]

Let \( f(x) = 2x + 1 \). Find \( f(-2) \), \( f(0) \), \( f(a) \), \( f(a + h) \), \( \frac{f(a + h) - f(a)}{h} \)

Let \( g(x) = 2 - x^2 \). Find \( g(-2) \), \( g(0) \), \( g(a) \), \( g(a + h) \), \( \frac{g(a + h) - g(a)}{h} \)
Function Notation and Operations of Functions

Do all work on notebook paper. All steps should be shown. All work should be neat and organized.

For the given functions, find the following values:
\( f(-2), f(-1), f(0), f(1), f(a), f(a+h) \)

1. \( f(x) = 2x - 3 \)
2. \( f(x) = 3 - x \)
3. \( f(x) = x^2 - 2 \)
4. \( f(x) = 5 - x^2 \)
5. \( f(x) = 2x^2 \)

Find \( (f + g)(x), (f - g)(x), (f \cdot g)(x), \left(\frac{f}{g}\right)(x) \).

6. \( f(x) = 2x^2 + 5x - 1, \quad g(x) = x - 2 \)
7. \( f(x) = x^2 + 5, \quad g(x) = x^2 - 9 \)
8. \( f(x) = 2x + 3, \quad g(x) = x - 11 \)
9. \( f(x) = x^3 + 3x - 5, \quad g(x) = 2x + 1 \)

Let \( f(x) = 3x - 1, \quad g(x) = 3x^2 + 5x - 1, \quad m(x) = x^2 - 4 \), and \( p(x) = 2x + 1 \). Find the following:

10. \( f(-2) \)
11. \( m(-3) \)
12. \( 5f(-2) + 4m(-3) \)
13. \( [f(-2)]^3 \)
14. \( g(3) \)
15. \( \left(\frac{m(-3)}{g(3)}\right)^2 \)
16. \( (m \cdot p)(-1) \)
17. \( (g - f)(0) \)
18. \( g(x) - m(x) \)
19. \( \frac{m(x)}{p(x)} \)
20. \( g(2x) \)
21. \( p(3x) \)
22. \( f(a) \)
23. \( f(a + h) \)
24. \( \frac{f(a + h) - f(a)}{h} \)
25. \( g(a) \)
26. \( g(a + h) \)
27. \( \frac{g(a + h) - g(a)}{h} \)
The slope-intercept formula of a line is: \( y = mx + b \)

Where \( m \) is the slope of the line and \( b \) is the \( y \)-intercept.

Solve for \( y \). Identify the slope and \( y \)-intercept.

Ex #1. \( 3x - 4y = 10 \)  
Ex #2. \( \frac{2}{3} x - \frac{1}{5} y = -3 \)  
Ex #3. \( 5y + 15 = 0 \)
Part 2: Find the $x$-intercept and $y$-intercept.
The $x$-intercept is where the graph crosses the $x$-axis and the $y$-intercept is where the graph crosses the $y$-axis.

Find the $x$-intercept and $y$-intercept.

Ex. #1: $4x + 3y = 8$

Ex. #2: $2x - 5y = 0$

Ex. #3: $y = 7x - 1$

Ex. #4: $\frac{3}{4}x - \frac{2}{5}y = -1$
Part 3: Find the slopes of the lines passing through the following points.

Formula for slope: \( m = \frac{y_2 - y_1}{x_2 - x_1} \)

Find the slopes of the lines passing through the following points.

Ex #1: (7,0) and (0,4)  
Ex #2: (−2,−5) and (1,9)

Ex #3: (3,−5) and (−1,−5)  
Ex #4: (7,−2) and (7,5)
Part 4: Graphing lines using the slope and y-intercept.
1. Solve the equation for \( y \).
2. Identify \( m \) and \( b \).
3. Plot \( b \) on the y-axis.
4. From \( b \), use the slope \( \left( \frac{\text{rise}}{\text{run}} \right) \) to get more points.

Ex. #1: \( 4x - y = -1 \)

Ex. #2: \( 2x + 3y = 9 \)

Ex. #3: \( y = 2x - \frac{7}{2} \)

Ex. #4: \( y = -2 \)

Ex. #5: \( 3x - 12 = 0 \)
Linear Functions

Do all work on notebook paper. All steps should be shown. All work should be neat and organized.

Solve for $y$. Identify the slope and $y$-intercept.

1. $2x - y = 6$
2. $3x + 4y = 8$
3. $x + 3y = 0$
4. $-2x = 3y + 8$
5. $4y - 12 = 0$
6. $\frac{1}{3}y + \frac{3}{4}x - 2 = 0$

Find the $x$-intercept and $y$-intercept.

7. $3x + 4y = 6$
8. $4x - 5y = 10$
9. $5x - 3y = 0$
10. $y = 2x + 5$
11. $\frac{2}{3}y - \frac{1}{5}x = -3$

Find the slopes of the lines passing through the following points.

12. $(6,0)$ and $(0,-3)$
13. $(-4,1)$ and $(3,-5)$
14. $(3,-7)$ and $(6,2)$
15. $(5,2)$ and $(9,2)$
16. $(-3,1)$ and $(-3,10)$
Graph the following lines. Find the slope and y-intercept.

17. $3x - y = 2$  \hspace{1cm} m= \hspace{1cm} b= 

18. $x + 2y = 4$  \hspace{1cm} m= \hspace{1cm} b= 

19. $4y - 3x = 8$  \hspace{1cm} m= \hspace{1cm} b= 

20. $3x + 5y = 0$  \hspace{1cm} m= \hspace{1cm} b= 

21. $y = -2x + \frac{5}{2}$  \hspace{1cm} m= \hspace{1cm} b= 

22. $y = 4$  \hspace{1cm} m= \hspace{1cm} b= 

23. $x = -3$  \hspace{1cm} m= \hspace{1cm} b= 

24. $3y + 6 = 0$  \hspace{1cm} m= \hspace{1cm} b= 

25. $5x - 15 = 0$  \hspace{1cm} m= \hspace{1cm} b=
1. Find the equation of a line given the slope and y-intercept.
2. Find the equation of a line given the slope and a point.

Find the equation of the line with the given information. Write answers in slope-intercept form, if possible.

You will need to know 2 formulas:
1. Slope-intercept formula: \( y = mx + b \)
2. Point-Slope formula: \( y - y_1 = m(x - x_1) \)

Ex. #1: \( m = \frac{2}{5} \); y-intercept = -5
Ex. #2: \( m = 0 \); y-intercept = \(-\frac{1}{2}\)

Find an equation of a line given a slope and a point:
Use the Point-Slope Formula: \( y - y_1 = m(x - x_1) \)

\( m \) = slope \quad Point \ (x_1, y_1)

Ex. #3: \( m = 5 \); through \((-2,1)\)
Ex. #4: \( m = -\frac{3}{5} \); through \((-4,-2)\)
Extra Practice: \( m = \frac{2}{3} \); through \((4, -1)\)

<table>
<thead>
<tr>
<th>Horizontal</th>
<th>Vertical</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equation: ( y = \text{number} ) ( m = 0 ) only has a ( y )-intercept</td>
<td>Equation: ( x = \text{number} ) ( m ) is undefined only has an ( x )-intercept</td>
</tr>
<tr>
<td>Ex. #5: ( m = 0 ); through ((-5, 3))</td>
<td>Ex. #6: ( m ) is undefined; through ((-2, -7))</td>
</tr>
</tbody>
</table>
Find the equation of the line passing through the given points.

1. Find the slope first. \[ m = \frac{y_2 - y_1}{x_2 - x_1} \]
2. Pick one point and now use the Point-Slope formula. \[ y - y_1 = m(x - x_1) \]
3. Write answers in slope-intercept form, if possible.

Ex. #1: Passing through the points (−1,3) and (4,7)

Ex. #2: Passing through the points (3,−4) and (−5,−1)

Ex. #3: Passing through the points (5,−6) and (−3,−6)

Ex. #4: Passing through the points (−7,−4) and (−7,8)
Equations of Lines

Do all work on notebook paper. All steps should be shown. All work should be neat and organized.

Find the equations of the following lines. Write answers in slope-intercept form.

1. \( m = 5; \) y-intercept \( = \frac{1}{2} \)
2. \( m = 1; \) y-intercept \( = -9 \)
3. \( m = \frac{1}{2}; \) y-intercept \( = -3 \)
4. \( m = 0; \) y-intercept \( = 11 \)
5. \( m = 0; \) y-intercept \( = \frac{2}{5} \)

Find the equations of the following lines. Write answers in slope-intercept form when possible.

6. \( m = 3; \) through \((-1, -2)\)
7. \( m = \frac{9}{2}; \) through \((3, 8)\)
8. \( m = -\frac{2}{3}; \) through \((5, 3)\)
9. \( m = 0; \) through \((4, -12)\)
10. \( m \) is undefined; through \((3, 7)\)

Find the equations of the lines passing through the given points. Write answers in slope-intercept form when possible.

11. \((-2, 4)\) and \((-5, 7)\)
12. \((-8, 6)\) and \((4, -3)\)
13. \((0, 0)\) and \((-2, 3)\)
14. \((8, -4)\) and \((-3, -4)\)
15. \((5, 3)\) and \((5, -6)\)
Parallel Lines have the same slope.
Find the equations of the lines passing through the given points parallel to the given line. Write answers in slope-intercept form when possible.

Ex #1: Through (2,5); parallel to $3x + 7y = 14$

1. Find the slope of the given line by solving for $y$.

2. Use the slope and the given point to write equation of line.

3. Write answers in slope-intercept form when possible.

Ex #2: Through $(-4, -9)$; parallel to $y = 2$

Ex #3: Through $(7, -2)$; parallel to $x = 8$
Perpendicular Lines slopes are opposite reciprocals. (flip and change the sign)
Find the equations of the lines passing through the given points perpendicular to the given line. Write answers in slope-intercept form when possible.

Ex #1: Through (2,5); perpendicular to \( y = 4x - 5 \)

1. Find the slope of the given line by solving for \( y \).
2. Find the opposite reciprocal of the slope. We label this \( m_\perp \).
3. Use \( m_\perp \) and the given point to write equation of line.
4. Write answers in slope-intercept form when possible.

Ex #2: Through (−7,2); perpendicular to \( 3x - 5y = 15 \)

Ex #3: Through (−4,−9); perpendicular to \( y = 8 \)

Ex #4: Through (7,−2); perpendicular to \( x = −3 \)
Parallel and Perpendicular Lines

Do all work on notebook paper. All steps should be shown. All work should be neat and organized.

Find the equations of the lines passing through the given points parallel to the given line. Write answers in slope-intercept form when possible.

1. Through \((1, -2)\); parallel to \(y = 3x + 4\)
2. Through \((3, 11)\); parallel to \(5x + 4y = -8\)
3. Through \((-2, 5)\); parallel to \(3x - 7y = 21\)
4. Through \((3, -7)\); parallel to \(y = 12\)
5. Through \((-8, -9)\); parallel to \(x = 5\)

Find the equations of the lines passing through the given points perpendicular to the given line. Write answers in slope-intercept form when possible.

6. Through \((-1, 2)\); perpendicular to \(y = 3x + 4\)
7. Through \((3, 11)\); perpendicular to \(3x - 5y = -10\)
8. Through \((-2, 5)\); perpendicular to \(4x + 3y = 21\)
9. Through \((3, -7)\); perpendicular to \(y = 4\)
10. Through \((-8, -9)\); perpendicular to \(x = -7\)
Graph the solution set of the linear inequalities:

Steps:
1. Solve for $y$. Identify the slope and $y$-intercept.
2. Graph the line by plotting the $y$-intercept first (on the $y$-axis) and then use the slope to get other points, $\left(\frac{\text{rise}}{\text{run}}\right)$.
3. Use a solid line if you have $\leq$ or $\geq$.
   Use a dashed or dotted line if you have $<$ or $>$. 
4. Look at the $y$-intercept. Shade below the $y$-intercept if less than. Shade above the $y$-intercept if greater than.

Ex. #1: $y - 3x > 2$  
Ex. #2: $9 > 4x + 3y$  
Ex. #3: $4x \leq 12 - 3y$  
Ex. #4: $x > -2$  
Ex. #5: $y \leq 4$
## Linear Inequalities in Two Variables

Solve for $y$, if possible. Identify the slope and $y$-intercept. Graph.

1. $y - 2x < 6$

   \[
   \begin{align*}
   y &= \_\_\_\_ \\
   m &= \_\_\_\_ \\
   b &= \_\_\_\_
   \end{align*}
   \]

2. $3x + 2y \geq 12$

   \[
   \begin{align*}
   y &= \_\_\_\_ \\
   m &= \_\_\_\_ \\
   b &= \_\_\_\_
   \end{align*}
   \]

3. $y \leq \frac{1}{3}x - 1$

   \[
   \begin{align*}
   y &= \_\_\_\_ \\
   m &= \_\_\_\_ \\
   b &= \_\_\_\_
   \end{align*}
   \]

4. $y > 4x - 5$

   \[
   \begin{align*}
   y &= \_\_\_\_ \\
   m &= \_\_\_\_ \\
   b &= \_\_\_\_
   \end{align*}
   \]

5. $4x \leq 2y - 6$

   \[
   \begin{align*}
   y &= \_\_\_\_ \\
   m &= \_\_\_\_ \\
   b &= \_\_\_\_
   \end{align*}
   \]

6. $7 < 3x - y$

   \[
   \begin{align*}
   y &= \_\_\_\_ \\
   m &= \_\_\_\_ \\
   b &= \_\_\_\_
   \end{align*}
   \]
<table>
<thead>
<tr>
<th></th>
<th>5x &gt; -3y + 9</th>
<th>2x &gt; 3y</th>
<th>x + 4y ≥ 0</th>
<th>y &lt; 3</th>
<th>x ≥ -5</th>
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<tr>
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<td>8</td>
<td>-2y ≤ 7x - 8</td>
<td>10. x + 4y ≥ 0</td>
<td>11. y &lt; 3</td>
<td>12. x ≥ -5</td>
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SYSTEMS OF EQUATIONS

Substitution Method, Elimination Method, Applications
SUBSTITUTION METHOD:
1. Choose one of the equations and solve for one of the variables. Try to pick a variable with a coefficient of 1 or –1. This will eliminate having a lot of fractions.
2. Substitute this expression into the OTHER equation.
3. Solve the resulting equation. (It should now have only one variable)
4. Take this solution and substitute it into the expression obtained in step one.
5. Write your answer as an ordered pair.

Ex. #1: Solve by the substitution method:
\[4x + y = 24\]
\[2x - 3y = -2\]

Ex. #2: Solve by the substitution method:
\[-x + 2y = 9\]
\[4x + 3y = -14\]
ELIMINATION METHOD:
1. If necessary, rewrite both equations in the form of \( Ax + By = C \).
2. Multiply either equation or both equations by appropriate numbers so that the coefficients of \( x \) or \( y \) will be opposites with a sum of \( 0 \).
3. Add the equations.
4. Solve this equation.
5. Substitute this solution back into one of the ORIGINAL equations.
6. Write your answer as an ordered pair.

Ex. #3: Solve by the elimination method:
\[
\begin{align*}
3x + 7y &= -1 \\
2x + y &= 3
\end{align*}
\]

Ex. #4: Solve by the elimination method:
\[
\begin{align*}
4x - 3y &= 25 \\
-3x + 8y &= 10
\end{align*}
\]
Linear Systems in Two Variables

Do all work on notebook paper. All steps should be shown. All work should be neat and organized.

Use the substitution method to determine the solution(s) of the following linear systems of equations.

1. \[2x - 3y = 7\]  \[y = x - 2\]
2. \[x + 3y = 11\]  \[2x - y = -6\]
3. \[x + 2y = 8\]  \[3x + y = 9\]
4. \[5x - y = -4\]  \[4x + 2y = 8\]
5. \[3x - 6y = 15\]  \[x - 2y = 11\]
6. \[6x - 2y = -14\]  \[3x - y = -7\]

Use the elimination method to determine the solution(s) of the following linear systems of equations.

7. \[2x - 3y = -1\]  \[x - 2y = 1\]
8. \[2x - y = 4\]  \[6x - 3y = 12\]
9. \[-5x + 2y = -32\]  \[4x - 3y = 27\]
10. \[4x + 5y = -3\]  \[-8x - 3y = 6\]
11. \[-3x + 2y = -2\]  \[-5x - 4y = 26\]
12. \[-3x + 2y = -7\]  \[6x - 4y = -1\]
13. \[5x - 2y = -18\]  \[-3x + 5y = 26\]
Solve word problems using a system of equations.

Ex. #1: The sum of two numbers is 26. The larger number is 5 more than twice the smaller number. What are the numbers?

Ex. #2: The difference of two numbers is 12. Twice the larger number plus the smaller number is 18. What are the numbers?

Money problems:

If I have 3 quarters, I have 75¢. \(3\text{(value of coin)} = \$\text{money}\)
If I have 9 nickels, I have 45¢. \(9\text{(value of coin)} = \$\text{money}\)

\((\text{How many of the item})(\text{Value of the item}) = \$\text{Total money}\)

Ex. #3: Jan has $5.90 in dimes and quarters. She has a total of 32 coins. How many dimes and how many quarters does Jan have?

Ex. #4: Elaine spent $5.96 on 12¢ and 26¢ stamps. She bought a total of 31 stamps. How many 12¢ stamps and how many 26¢ did she buy?
Solve word problems using a system of equations.

Ex. #1: Alex sold 32 tickets to the high school football game. Adult tickets cost $4.00 and student tickets cost $1.50. Alex collected $103 from the sale of the tickets. How many adult tickets and how many student tickets did Alex sell?

1. Define the variables.
2. Write the equations.
3. Solve.
4. Answer the problem.

Ex. #2: Yolanda bought 3 pens and 1 notebook for $1.70. Four days later she bought 2 pens and 2 notebooks for $2.36. What is the cost of each pen and each notebook?
Applications-System of Equations

Do all work on notebook paper. All steps should be shown. All work should be neat and organized.

Define the variable. Write a system of equations. Solve.

1. The sum of two numbers is 25. The larger number is 1 more than 5 times the smaller number. What are the numbers?

2. The sum of two numbers is 6, and their difference is 10. What are the numbers?

3. The difference of two numbers is 10. Twice the larger number minus the smaller number is 14. What are the numbers?

4. Juanita has $2.80 in nickels and dimes. She has a total of 40 coins. How many nickels and how many dimes does Juanita have?

5. Shana spent $5.28 on 20¢ and 32¢ stamps. She bought a total of 21 stamps. How many 20¢ stamps and how many 32¢ stamps did she buy?

6. Devon sold 37 tickets to the choir show. Adult tickets cost $3.50 and children’s tickets cost $1.25. He collected $73.25 from the sale of the tickets. How many adult tickets and how many children’s tickets did he sell?

7. Maritza bought 4 pencils and 1 eraser for 62¢. The next week she bought 3 pencils and 7 erasers for 84¢. What is the cost of each pencil and eraser?

8. Coach Reeves buys 3 basketballs and 2 footballs for $60. Coach Johnson buys 2 basketballs and 5 footballs for $73. What is the cost of each basketball and football?
ABSOLUTE VALUE
Absolute value of a real number is the _____________ from zero on a number line. Distance is always positive.

\[ |x| = 2 \]

\[ |x| = -3 \]

\[ |x| = 0 \]

Process:
1. Isolate the absolute value expression. \[ |\text{expression}| = \text{number} \]
2. Determine the type of number the absolute value expression is equal to
   a. If it is equal to a NEGATIVE NUMBER the answer is NO SOLUTION.
   b. If it is equal to a POSITIVE NUMBER you will split into 2 equations (without absolute value bars)—(2 solutions)
      \[ \text{expression} = \text{number} \quad \text{or} \quad \text{expression} = -(\text{number}) \]
   c. If it is equal to ZERO rewrite the equation without absolute value bars and solve for the variable. (one solution)

Example 1: \[ |2x + 5| = 13 \]
Example 2: $|3x - 4| - 3 = 11$

Example 3: $2|2x - 5| + 5 = 11$

Example 4: $|2x + 1| + 4 = 4$

Example 5: $2|2x - 5| + 9 = 9$

Example 6: $4|x - 1| + 7 = 3$

Example 7: $5 - |2x - 3| = 7$
Absolute Value Equations

Do all work on notebook paper. All steps should be shown. All work should be neat and organized.

Solve the equations.

1. $|x| = 8$
2. $2|x| = 12$
3. $|2x - 1| = 7$
4. $|3 - 4x| = 11$
5. $|3x - 8| = -7$
6. $\left| \frac{3}{4}x - 2 \right| = 4$
7. $|2x + 5| = 0$
8. $3|5x + 1| - 1 = 5$
9. $3|3x + 8| + 2 = 8$
10. $2|3x - 1| + 7 = 7$
11. $5|x - 4| + 6 = 1$
12. $5 - |2x - 3| = -7$
Absolute Value Inequalities

Recall that absolute value is the distance away from zero. The distance is ALWAYS positive.

\[ |x| < 4 \quad \text{and} \quad |x| \geq 3 \]

**Less Than: \((\leq, <)\)**
1. Isolate the absolute value: \(|\text{expression}| < \text{number}\)
2. RE-write without the absolute value sign. Use a "sandwich" inequality:
   \[-\text{number} < \text{expression} < \text{number}\]
3. Solve.
4. Graph answer on a number line.
5. Write answer in interval notation.

**Greater Than: \((\geq, >)\)**
1. Isolate the absolute value: \(|\text{expression}| > \text{number}\)
2. RE-write without the absolute value sign. You must separate into 2 inequalities:
   \[\text{expression} > \text{number} \quad \text{or} \quad \text{expression} < -\text{number}\]
3. Solve.
4. Graph answers on a number line.
5. Write answer in interval notation.

Example 1: \( |x - 4| \leq 8 \)  
Example 2: \( |x + 2| > 9 \)
Example 3: \(|3x - 1| - 2 \geq 9\)  \hspace{5cm} \text{Example 4: } \ 2|x + 3| + 5 < 11

Example 5: \(|5x - 13| + 7 \leq 6\)  \hspace{5cm} \text{Example 6: } \ 3|x + 1| + 5 > 1
**Absolute Value Inequalities**

Do all work on notebook paper. All steps should be shown. All work should be neat and organized.

Solve the inequalities. Graph solutions on a number line. Write the answers in interval notation.

1. $|x| < 3$
2. $|x| \geq 2$
3. $|x| + 2 \leq 6$
4. $|x| - 9 > 5$
5. $|x| \leq -11$
6. $|x| \geq -10$
7. $|x - 5| < 1$
8. $|2x - 5| > 3$
9. $|2x + 2| - 2 < 6$
10. $|2x - 1| + 1 \geq 8$
11. $|4x - 7| < 2$
12. $|3x - 5| + 12 < 6$
13. $|4x + 1| + 6 > 3$
14. $4|x - 1| - 7 > 13$
15. $3|x - 1| + 7 \leq 10$
FACTORYING

GCF, Grouping, Binomials, Trinomials, Solving Factorable Quadratic Equations
Greatest Common Factor

**GCF:** The largest number that will divide into **all** the terms AND the variable(s) that occur in **all** the terms (raised to the smallest power).

Find the GCF of the following polynomials:
1. $14x^3 - 21x^2 + 28x^4$
2. $-16a + 24b + 8a^2$

Factor the following:
3. $8x^2 + 18xy + 2x$
4. $-7x^4y - 35x^2y^2 - 14x^2y^5$

5. $7y(3x + 9) - 14z(3x + 9)$
6. $15x(y - 3) - 5z(3 - y)$
GROUPING: This factoring technique is used when you have 4 or more terms.

Try grouping the first two terms and the last terms.
Factor out the GCF of each group.
If the GCF matches exactly, then “take out” that GCF from both groups.
Factor further, if necessary.

Factor the following:
1. $10hn - 3km + 2hm - 15kn$

2. $-42x^2z^3 + 6x^2z^2 - 2xz^3 + 14xz^4$

3. $7xy^3 - 20xy + 2y^3 - 70x^2y$
GCF and Grouping

Do all work on notebook paper. All steps should be shown. All work should be neat and organized.

Factor completely.

1. $10x^2 - 35$
2. $8x^3 + 18xy + 2x$
3. $-6x^5 - 18x^3 - 10x^2$
4. $4x(x - 2y) - 6y(x - 2y)$
5. $2x(x - 3) + (x - 3)$
6. $8y^2z(1 - 5x) - 6yz^3(1 - 5x)$
7. $15x(y - 3) - 5z(3 - y)$
8. $2ax + 3bx + 2ay + 3by$
9. $6cf - 2cg + 6df - 2dg$
10. $4x + 15xy - 3x^2 - 20y$
11. $3x^2y - 5x^2y^2 - 12xy^2 + 20xy^3$
12. $3y^3 + 7y^2 + 18y + 42$
13. $8x^6 + 12x^4 + 40x^3 + 60x$
14. $54y^3 + 6x^4 - 27x^3y - 12xy^2$
**Notes**--  Factoring: Difference of 2 Squares

**ALWAYS look for a GCF first.**

Difference of 2 Squares: \( a^2 - b^2 = (a + b)(a - b) \)

1. \( 25x^2 - 81y^2 \)  
   2. \( 16x^4 - 1 \)

3. \( 2x^2 - 50 \)  
   4. \( (x + 2)^2 - 9 \)

5. \( (3x - 2y)^2 - 49z^2 \)  
   6. \( x^2(5x + 4) - 9(5x + 4) \)
Cubes: 1, 8, 27, 64, 125, 216, 1000

Sum of 2 cubes: \(a^3 + b^3 = (a + b)(a^2 - ab + b^2)\)

Difference of 2 cubes: \(a^3 - b^3 = (a - b)(a^2 + ab + b^2)\)

Sum of 2 cubes: \(x^3 + 8 = \)

Difference of 2 cubes: \(x^3 - 125 = \)

**ALWAYS look for a GCF first.**

1. \(125x^3 - 27y^3 = \)

2. \(40x^3 + 135y^3 = \)

3. \(y^4 - 2y^3 - 64y + 128 = \)

4. \(12x^5y - 75x^3y - 12x^3y^3 + 75xy^3 = \)
Factoring Binomials

Do all work on notebook paper. All steps should be shown. All work should be neat and organized.

Factor completely.

1. \( x^2 - 9 \)
2. \( 36x^2 - 49y^2 \)
3. \( 27x^3y - 12xy^3 \)
4. \( 3x^2 + 48 \)
5. \( 81x^4 - 16y^4 \)
6. \( (x-3)^2 - 49 \)
7. \( (2x+3y)^2 - 9z^2 \)
8. \( x^3 - 64 \)
9. \( 8x^3 + 1 \)
10. \( 64x^3 - 27y^3 \)
11. \( 54x^3 + 128y^3 \)
12. \( 375x^3 - 3 \)
13. \( x^2(2x-3) - (2x-3) \)
14. \( x^3(x-6) - 8(x-6) \)
15. \( 5x^3 + 2x^2 - 45x - 18 \)
16. \( 3x^3 + 4y^3 - 4x^2y - 3xy^2 \)
17. \( 3x^4 - 5x^3 - 3x + 5 \)
18. \( 4x^4 - 3x^3 + 32x - 24 \)
19. \( 4x^5 - x^3 - 32x^2 + 8 \)
20. \( 7x^5 - 7x^3 - 28x^3y^2 + 28xy^2 \)
TRINOMIAL: A polynomial with 3 terms.

ALWAYS look for a GCF first.
It is a good idea to make leading coefficient positive.
Guess and check method is one of many methods, but not the only one.

Factor the following:
1. \( x^2 - 14x + 45 \)  
2. \( x^2 + 6x + 10 \)

3. \( -8x^2 + 16x - 40 \)  
4. \( 6x^2 + 7x - 20 \)

5. \( 2x^2 - 5x - 12 \)  
6. \( 24 - 22x + 3x^2 \)

7. \( 8x^2 + 6xy - 5y^2 \)  
8. \( x^4 + 4x^2 - 5 \)
Factoring Trinomials

Do all work on notebook paper. All steps should be shown. All work should be neat and organized.

Factor completely.

1. \(x^2 - 9x + 14\)
2. \(x^2 - 9x + 8\)
3. \(x^2 - x + 7\)
4. \(x^2 - 6x + 9\)
5. \(3x^2 + 24x + 21\)
6. \(-2x^3 - 16x^2 - 32x\)
7. \(x^2 + 6xy + 5y^2\)
8. \(6x^2 - 13x - 5\)
9. \(12x^2 + 25x - 7\)
10. \(9x^2 - 15x + 4\)
11. \(9x^2 + 30x + 25\)
12. \(-2 + 17x + 9x^2\)
13. \(6x^2 - 13x - 15\)
14. \(20x^2 - 60x - 35\)
15. \(15x^3 + 42x^2 - 9x\)
16. \(9x^2 - 16xy - 4y^2\)
17. \(x^4 + 2x^2 - 3\)
18. \(4x^4 + 7x^2 - 2\)
19. \(6x^2 + 19x + 15\)
20. \(10x^2 - 19x + 6\)
A quadratic equation has the form of \( ax^2 + bx + c = 0 \)

Steps to solve by factoring:
1. Write in standard form: \( ax^2 + bx + c = 0 \)
2. Factor.
3. Set each factor that has a variable equal to zero.
4. Solve each resulting linear equation.

1. \(-14x^2 + 21x = 0\)
2. \(12x^2 - 26x - 10 = 0\)

3. \(5x^2 + 8x - 1 = x(x + 8)\)
4. \(7x^2 + 23x + 4 = (x + 4)(x - 4)\)
Quadratic Equations

Do all work on notebook paper. All steps should be shown. All work should be neat and organized.

Solve by factoring.

1. \( x^2 + 6x + 8 = 0 \)
2. \( 2x^2 + 12x - 32 = 0 \)
3. \( 6x^2 - 2x = 0 \)
4. \( 4x^2 - 9 = 0 \)
5. \( 2x^2 - 7x - 4 = 0 \)
6. \( 6x^2 - 41x - 7 = 0 \)
7. \( 12x^2 - 31x + 9 = 0 \)
8. \( x^2 + 5x - 4 = 10 \)
9. \( 8x^2 - 13x - 9 = 5x - 4 \)
10. \( 3x^2 + 5x + 20 = 2x^2 - 3x + 4 \)
11. \( 9x^2 + 7x - 7 = 3x(x - 4) \)
12. \( 3x^2 - 8x - 31 = (2x + 3)(x - 7) \)
13. \( (2x + 5)(x - 4) = (x + 7)(x - 4) \)
RATIONAL EXPRESSIONS

Integer Exponents, Reducing Expressions, Performing Operations on Rational Expressions, Complex Fractions, Long Division, Synthetic Division, Solving Equations
Evaluating Integer Exponents

\[ 5^2 \]

**Negative Exponents:** \(2^{-3}\)

Take the reciprocal of base and change the sign of the exponent.

\[
\frac{x^{-2}}{y^3} = \quad \frac{x^{-3}}{y^{-2}} =
\]

**Zero Exponent:** \(5^0 = x^0 = (10x^2y^4)^0 =
\]

ANYTHING (except zero) raised to the power of zero equals 1.

Evaluate each of the following.

1. \(2^5 \cdot 2^{-2}\)
2. \(-5^{-4}\)
3. \((-2)^{-4}\)

4. \((-5)^{-2}\) compared to \(-5^{-2}\)

5. \(3^{-1} - 2^{-2}\)
6. \(3^{-2} - 3^{-1}\)

7. \((3^{-2} - 3^{-1})^{-2}\)
8. \(-5^{-1} + 5^0 + 5\)
Review Negative Exponents:
Take the reciprocal of base and change the sign of the exponent.

\[
\frac{x^{-2}}{y^3} = \quad \frac{x^{-3}}{y^2} = 
\]

Zero Exponent:

\[
x^0 = \quad (10x^2y^4)^0 = 
\]

Simplify each of the following. Your answers should have no NEGATIVE exponents.

1. Multiply the exponents to get rid of parentheses.
2. Make all exponents positive.
3. Clean up.

1. \( (3x^{-2}y^{-3})^2 (5xy^{-1})^{-3} \)
2. \( \frac{(3x^5y^{-4})^{-2}}{(2x^{-2}y^{-3})^3} \)
3. \( \left( \frac{5x^2y^{-5}}{8x^6y^{-12}} \right)^2 \)
4. \( \left( \frac{6x^3y^{-2}}{9^0x^{-5}y^{-6}} \right)^{-3} \)
Evaluate each of the following. Your answers should have NO exponents.

1. \(2^2 \cdot 2^3\)
2. \(\frac{(-7)^4}{(-7)^5}\)
3. \((4^{-1})^{-2}\)
4. \(-8^2\)
5. \((-8)^2\)
6. \(-2^3\)
7. \((-2)^3\)
8. \((-3)^{-4}\)
9. \(-3^{-4}\)
10. \((-3)^{-3}\)
11. \(-3^{-3}\)
12. \(2^{-3} + 4^{-1}\)
13. \(4^{-2} + 3^{-1}\)
14. \(2^{-2} + 4^{-1}\)
15. \(8^0 + 8^{-1} + 8\)
16. \(5^0 + 5^{-1} + 5\)
17. \(\left(\frac{2}{3}\right)^3\)
18. \(\left(\frac{5}{8}\right)^{-1}\)
19. \(\left(\frac{4}{3}\right)^{-2}\)
20. \(\frac{5^{-2}}{3^{-1}}\)

Simplify each of the following. Your answers should have no NEGATIVE exponents.

21. \(\left(\frac{8x^9 y^3}{4x^3 y^2}\right)^2\)
22. \((3x^{-2} y^2)^{-1} (4xy^3)^{-2}\)
23. \((8x^0 y^{-4})^0\)
24. \(\left(\frac{2x^4}{5y^{-2}}\right)^3\)
25. \(\left(\frac{5x^{-3} y}{3x^2 y^{-2}}\right)^3\)
26. \(\frac{8x^3 y^{-2}}{10x^6 y^5}\)
27. \(\frac{2^{-2} x^{-3} y^{-4}}{8^{-1} x^{-1} y^{-2}}\)
28. \(\frac{(2x^3 y^{-4})^4}{(5x^0 y^7)^2}\)
29. \(\frac{(-3x^4 y^7)^3}{(2x^{-4} y^3)^4}\)
30. \(\frac{(4^0 x^{-2} y^6)^{-2}}{8x^3 y^4}\)
Reducing Rational Expressions

Review Reducing Fractions:

\[
\frac{25}{30} = \frac{7}{9} = \frac{x + 5}{x^2 + 25}
\]

1. Factor.
2. “Cancel” common FACTORS.

1. \[\frac{8xy^5z^3}{-2x^3y^4z}\]
2. \[\frac{2x^2y - 14xy}{x^2y + 3xy^2}\]
3. \[\frac{x^2 - 6x - 16}{64 - x^2}\]
4. \[\frac{8x^3 + 27}{2x^2 - 3x - 9}\]
Reducing Rational Expressions

Do all work on notebook paper. All steps should be shown. All work should be neat and organized.

Reduce the following expressions to lowest terms.

1. \( \frac{8x^4y^2}{6xy^3} \)
2. \( \frac{-9ab^2c^3}{4ab^3c} \)
3. \( \frac{3x^3 + x^2y}{x^2y^2 - 2xy^3} \)
4. \( \frac{3x + 6y}{3x} \)
5. \( \frac{4x^3y - 4x^2y}{2xy^2 - 2x^2y^2} \)
6. \( \frac{x^2 + 5x + 6}{x^2 - 4x - 21} \)
7. \( \frac{6x^2 + 7x - 3}{4x^2 - 9} \)
8. \( \frac{6x^2 + 7x - 5}{2x^2 - x} \)
9. \( \frac{x^2 + 3x - 10}{4 - x^2} \)
10. \( \frac{4x^2 - 12x + 9}{2x^2 - 11x + 12} \)
11. \( \frac{3x^2 + 9x - 30}{6x^2 + 30x} \)
12. \( \frac{2x^2 + 5x - 12}{9 - 4x^2} \)
13. \( \frac{x^3 + 8}{3x^2 + 2x - 8} \)
14. \( \frac{27x^3 - 1}{6x^2 + 4x - 2} \)
Directions:
Perform the indicated operations and reduce to lowest terms.
Even though the "indicated operations" are multiplication and division, what we need to do is **FACTOR** and **REDUCE**.

1. \[
\frac{10m^6p^7}{9x^3y^2} \cdot \frac{18x^5y^3}{3m^3p}
\]

2. \[
\frac{4-x}{6x+9} \div \frac{5x-20}{2xz+3z}
\]

3. \[
\frac{2x^2 - x - 15}{2x^2 + 9x + 10} \cdot \frac{x^2 - 3x - 10}{21x - 7x^2}
\]
Multiplying and Dividing Rational Expressions

Do all work on notebook paper. All steps should be shown. All work should be neat and organized.

Perform the indicated operations and reduce to lowest terms.

1. \[
\frac{5xy^2}{7a^3b^4} \div \frac{3x^4y^7}{6a^2b^6} \div \frac{14x^5y^2}{15a^4b^3}
\]

2. \[
\frac{5x^3y^5}{8a^7b^2} \div \frac{14ab^7}{9xy^8} + \frac{a^2b^2}{12xy}
\]

3. \[
\frac{2x+2}{5x-15} \div \frac{x-3}{xy+y}
\]

4. \[
\frac{10x^2y^2 + 5xy^2}{8-4x} \div \frac{6x+3}{12x-24}
\]

5. \[
\frac{10x^2 + 17x + 3}{2x^2 + 7x + 6} \div \frac{5x^2 + 41x + 8}{2x^2 + x - 6}
\]

6. \[
\frac{x^2 - 1}{3x^2 - x - 4} \div \frac{3x^2 + 2x - 8}{5x - 5x^2}
\]

7. \[
\frac{6 - 23x - 4x^2}{3x^2 + 10x - 48} \div \frac{3x^2 + x - 24}{8x^2 - 6x + 1}
\]

8. \[
\frac{3x^2 - 10x - 8}{2x^2 - 3x - 20} \cdot (2x+5)
\]

9. \[
\frac{x^3 - 8}{x^2 + 2x - 3} \div \frac{3x^2 - 2x - 1}{3x^2 - 5x - 2}
\]

10. \[
\frac{x^3 - 1}{2x^2 - 11x + 12} \div \frac{2x^2 - 3x - 20}{8x^3 + 12x^2} \div \frac{2x^2 + 3x - 5}{4x^2 - 9}
\]
When adding or subtracting fractions that have the same denominator:
1. Keep the denominator and collect like terms in the numerators.
2. Try to reduce the expression by factoring.

Directions:
Perform the indicated operations and reduce to lowest terms.

1. \( \frac{25}{3x} - \frac{4}{3x} \)
2. \( \frac{3x + 1}{5x - 2} + \frac{4x - 6}{5x - 2} \)
3. \( \frac{x^2}{2x + 3} - \frac{x + 6}{2x + 3} \)

OPPOSITE DENOMINATORS

4. \( \frac{5}{x - 2} - \frac{6}{2 - x} \)
5. \( \frac{x}{y - 3} + \frac{5x}{3 - y} \)

6. \( \frac{3x^2 - 7}{x - 4} + \frac{x^2 - 2x + 12}{4 - x} \)
When adding or subtracting fractions that have different denominators, one must find the least common denominator (LCD) before adding or subtracting the fractions.

Process to find the LCD:
1. Factor each denominator.
2. Write down one of every kind of factor.
3. Raise each factor to its highest power.

Find the LCD:

\[
\frac{5}{a^2b^3c} - \frac{7}{ab^4c^5d}
\]

\[
\frac{5}{6x^3y} + \frac{7}{4x^2y^5}
\]

Perform the indicated operations and reduce to lowest terms:

1. \[\frac{5}{x^3} - 1 \quad \frac{3}{x-1} + \frac{6}{x+4}\]
2. \[\frac{x-4}{3x-4} \quad \frac{2x-7}{x^2+3x-4} + \frac{x+31}{x^2-x-20}\]
3. \[\frac{3x-2}{2x^2-9x+10} - \frac{x+6}{x^2-6x+8}\]
Adding and Subtracting Rational Expressions

Do all work on notebook paper. All steps should be shown. All work should be neat and organized.

Perform the indicated operations and reduce to lowest terms.

1. \( \frac{3}{2x} + \frac{7}{2x} \)

2. \( \frac{2n}{xy} - \frac{9n}{xy} \)

3. \( \frac{x}{4x-7} - \frac{3x+5}{4x-7} \)

4. \( \frac{5x+1}{2x^2 + x - 11} - \frac{2x+9}{2x^2 + x - 11} \)

5. \( \frac{x^2 + 2x}{x+1} + \frac{1}{x+1} \)

6. \( \frac{7x}{y-2} - \frac{3x}{2-y} \)

7. \( \frac{2x^2 - 8}{x-5} + \frac{x^2 + x + 12}{5-x} \)

8. \( \frac{2x^2 - 5}{x-1} - \frac{x^2 + 2}{1-x} \)

9. \( \frac{8}{5x} + \frac{3}{2y} \)

10. \( \frac{3}{4xy^2} + \frac{7}{3x^3y} \)

11. \( \frac{5}{x+2} + \frac{2}{x+3} \)

12. \( \frac{3}{2x+5} - \frac{5}{x-3} \)

13. \( \frac{x+2}{2x-3} - \frac{x-3}{x+6} \)

14. \( \frac{x-1}{2x+5} + \frac{3x-2}{2x-3} \)

15. \( \frac{2}{x^2 - 4x - 5} + \frac{5}{x^2 - 2x - 15} \)

16. \( \frac{4x}{3x^2 - 5x - 2} - \frac{1}{3x^2 + 13x + 4} \)

17. \( \frac{2x+3}{x^2 - 1} + \frac{x-2}{x^2 - 6x + 5} \)

18. \( \frac{3x-5}{x^2 - x - 12} - \frac{x+1}{x^2 + 5x + 6} \)

19. \( \frac{x-1}{6x^2 - 7x + 2} + \frac{x+2}{2x^2 - 7x + 3} \)

20. \( \frac{x+1}{4x^2 + 4x - 15} - \frac{4x+5}{8x^2 - 10x - 3} \)
Synthetic Division is a condensed method of long division. It is quick and easy. Unfortunately, it can only be used when the divisor is in the form of \((x \pm a)\).

Review long division: \[
\frac{9x^2 - 5x + 1}{x - 1}
\]

Synthetic division:
\[
\begin{array}{c}
9x^2 - 5x + 1 \\
\hline
x - 1 \\
32253 \\
\hline
1
\end{array}
\]

\[
\begin{array}{c}
2x^3 - 5x^2 - 3 \\
\hline
x + 1 \\
42510 \\
\hline
312
\end{array}
\]

\[
\begin{array}{c}
x^4 - 5x^2 + 10x \\
\hline
x + 3 \\
x^2 - 2x + 10 \\
\hline
x + 3
\end{array}
\]

\[
\begin{array}{c}
x^3 + 125 \\
\hline
x + 5 \\
x^2 - 5x + 25 \\
\hline
x + 5
\end{array}
\]

Reminders:
1. Write both polynomials in standard form.
2. Fill in all missing terms with a place holder of zero.
3. Write your answer as a polynomial that is **one degree less** than the dividend (numerator).
**Division of Polynomials**

Do all work on notebook paper. All steps should be shown. All work should be neat and organized.

Perform the indicated divisions using LONG division method.

1. \( \frac{24x^2 - 62x + 36}{4x - 7} \)  
5. \( \frac{4x^2 - 7}{2x - 3} \)

2. \( \frac{40x^2 - 44x + 17}{5x - 3} \)  
6. \( \frac{4x^3 - x - 7}{2x - 3} \)

3. \( \frac{2x^3 - 13x^2 - 10x + 19}{2x + 3} \)  
7. \( \frac{6x^3 + 20x^2 - 19}{2x + 6} \)

4. \( \frac{-19 - 8x + 21x^2 + 12x^3}{4x + 7} \)  
8. \( \frac{2x^4 + x^3 - 3x^2 + 9x - 13}{x^2 + x - 3} \)

Perform the indicated divisions using SYNTHETIC division.

9. \( \frac{x^2 - 3x + 5}{x - 2} \)  
13. \( \frac{x^4 - 2x^3 - 5x^2 + x + 5}{x + 2} \)

10. \( \frac{2x^3 - 7x^2 - x + 6}{x - 3} \)  
14. \( \frac{2x^4 - 3x^3 + x^2 + 7x - 5}{x - 1} \)

11. \( \frac{x^3 - 7x - 4}{x + 2} \)  
15. \( \frac{6x^4 - 7x^3 - 11x^2 + 2x + 3}{x - \frac{1}{6}} \)

12. \( \frac{3x^3 + 11x^2 - 5x}{x + 4} \)  
(Hint: need a place holder for the constant)
Notes-- **Complex Fractions**

A complex fraction is a fraction that has fractions in the numerator and/or the denominator.

Directions: Simplify the following fraction.

1. \( \frac{21x^4}{8y} \cdot \frac{7x^3}{16y^2} \)

2. \( \frac{x^2 - 5x + 6}{10x + 5} \cdot \frac{4 - x^2}{6x + 3} \)

3. \( \frac{3}{x} - \frac{1}{6} \cdot \frac{x}{y} \)

4. \( \frac{1}{5} \cdot \frac{x}{2} \cdot \frac{7x}{14xy} \)

**Single Fractions:**

Change the division to multiplication. Reduce.

**Multiple Fractions:**

1. Find the LCD of all the fractions.

2. Multiply every term by the LCD.

3. Reduce
5. \[
\frac{2}{3x} - \frac{7}{3x^2} - \frac{10}{8} - \frac{5}{x^2}
\]

6. \[
\frac{2}{x} + \frac{1}{y}
\]

7. \[
\frac{5 + h}{3 + h} - \frac{5}{3}
\]

Multiple Fractions:

1. Find the LCD of all the fractions.

2. Multiply every term by the LCD.

3. Reduce
Complex Fractions

Do all work on notebook paper. All steps should be shown. All work should be neat and organized.

Simplify the following fractions.

1. \[ \frac{5}{7} \div \frac{45}{16} \]
2. \[ \frac{3x}{2y^2} \div \frac{5x^3}{4y^3} \]
3. \[ \frac{2x^2 + 3x}{y^2 - y} \div \frac{8x^4 + 12x^3}{3y^3 - 3y^2} \]
4. \[ \frac{8xy - 4y}{6xy^2} \div \frac{12x^2y - 6xy}{9x^2y^2} \]
5. \[ \frac{2 + \frac{3}{x}}{1 - \frac{7}{x}} \]
6. \[ \frac{\frac{x}{4} \div \frac{y}{x^2}}{y^2} \]
7. \[ \frac{\frac{1}{x} \div \frac{1}{y}}{x^2 - y^2} \]
8. \[ \frac{1 - \frac{5}{2x} - \frac{3}{2x^2}}{2 - \frac{15}{2} + \frac{9}{2x^3}} \]
9. \[ \frac{2x - y}{2} \div \frac{1}{y - x} \]
10. \[ \frac{1 - \frac{1}{9x^2}}{1 - 3x} \]
11. \[ \frac{3}{x^2} \div \frac{1}{x - 3} \]
12. \[ \frac{1}{x} \div \frac{1}{x - y} \]
13. \[ \frac{4}{2 + \frac{h}{h}} \]
14. \[ \frac{4}{\frac{2x + 1}{5}} \div \frac{3}{2x + 1 - 2} \]
Rational Equations

1. Find the L.C.D.
2. Multiply **EVERY** term by the LCD to get rid of all the fractions.
   (OR Cross-multiply, if you can**)
3. Solve the resulting equation.
4. Check for extraneous solutions. (Substitute answers into the denominator to see if this would cause division by zero. You must throw out any solutions that cause division by zero because it is undefined.)

1. \[
\frac{7}{6x} - \frac{5}{8x} = \frac{13}{12}
\]

2. \[
\frac{1}{x^2} - \frac{1}{12x} = \frac{1}{12}
\]

3. \[
\frac{3}{x-1} + \frac{4}{x+2} = \frac{16}{x^2 + x - 2}
\]

4. \[
\frac{3}{x-2} - \frac{2}{x} = \frac{6}{x^2 - 2x}
\]
5. \( \frac{3}{x+1} = \frac{2x+5}{x^2 + x} \) **

6. \( \frac{5}{5x+3} = \frac{5x-2}{x-1} \) **

7. \( \frac{2x+4}{x+2} - \frac{x+3}{x-2} = \frac{1}{2} \)
Rational Equations

Do all work on notebook paper. All steps should be shown. All work should be neat and organized.

Solve the following equations. Do not forget to check solutions.

1. \( \frac{-3}{2x} = \frac{9}{8} \)
2. \( \frac{3}{2x} + \frac{1}{3x} = \frac{11}{12} \)
3. \( \frac{x+1}{2} = \frac{x-5}{4} \)
4. \( \frac{2}{2x-1} + \frac{3}{x-4} = \frac{5}{2x-1} \)
5. \( \frac{2}{x+1} - \frac{1}{3x-2} = \frac{3}{3x-2} \)
6. \( \frac{5}{2x+1} - \frac{1}{2x-1} = \frac{6}{4x^2 - 1} \)
7. \( \frac{1}{x} - \frac{2}{4x+1} = \frac{1}{8x^2 + 2x} \)
8. \( \frac{2x+1}{x-2} = \frac{2x+1}{x+2} + \frac{10}{x^2 - 4} \)
9. \( \frac{1}{x^2} - \frac{1}{6x} = \frac{1}{6} \)
10. \( \frac{5}{x^2} - \frac{1}{3x} = 2 \)
11. \( \frac{2}{2x-1} + \frac{x}{x+4} = \frac{36}{2x^2 + 7x - 4} \)
12. \( \frac{4x}{x-1} - \frac{x}{x+3} = \frac{-12}{x^2 + 2x - 3} \)
13. \( \frac{3x+6}{2x+5} = \frac{-1}{x} \)
14. \( \frac{x+3}{9x+1} = \frac{x+1}{7} \)
15. \( \frac{9x+3}{10x-30} = \frac{x}{x-3} \)
16. \( \frac{2}{x+3} + \frac{4}{x+1} = \frac{4}{3} \)
17. \( \frac{1}{x-5} - \frac{5}{x-2} = \frac{3}{2} \)
18. \( \frac{x+2}{2x-1} + \frac{x+5}{x+3} = \frac{5}{3} \)
**Consecutive integers**: Remember that integers are only negative and positive “whole numbers”. They do not include any decimals or fractions.

Two consecutive integers: $x, x+1$

Two consecutive **odd** integers: $x, x+2$

Two consecutive **even** integers: $x, x+2$

**Reciprocals**: If the number is $x$, then its reciprocal would be $\frac{1}{x}$.

**Define the variable. Write an equation. Solve.**

1. The sum of a number and its reciprocal is $\frac{29}{10}$. What is the number?

2. The difference of the reciprocals of two consecutive integers is $\frac{1}{2}$. What are the integers?
3. The sum of the reciprocals of two consecutive even integers is \( \frac{9}{40} \). What are the integers?

Work:

\[
\frac{1}{\text{time alone}} + \frac{1}{\text{time alone}} = \frac{1}{\text{time together}}
\]

4. Paris can wash her car in 4½ hours. Her friend, Celia, can wash the same car in 7 hours. Working together, how long will it take them to wash the car?

5. Working together, Joseph and Dylan can write the computer program in 11 hours. Working alone, Joseph can write the computer program in 15 hours. How long does it take Dylan to write the program by himself?
Applications-Rational Equations

Do all work on notebook paper. All steps should be shown. All work should be neat and organized.

Find the solutions to the following problems.

1. The sum of a number and its reciprocal is \( \frac{25}{12} \). What is the number?

2. The sum of the reciprocals of two consecutive integers is \( \frac{7}{12} \). What are the integers?

3. The sum of the reciprocals of two consecutive even integers is \( \frac{7}{24} \). What are the integers?

4. Tameka can mow her yard in 2 hours. Her brother Dante can mow the yard in 3 hours. Working together, how long will it take them to mow the yard?

5. Brittany can prepare her report in \( 1 \frac{1}{2} \) hours. Her co-worker, Firza, can prepare the report in 6 hours. Working together, how long will it take them to prepare the report?

6. Working together Debbie and Jim can clean the house in 4 hours. Working alone, Debbie can clean the house in 5 hours. How long does it take Jim to clean the house?

7. Working together, Elmer and his son can paint their house in 3 days. Working alone, Elmer can spray paint the house in 4 days. How long will it take his son, using a brush, to paint the house by himself?
RADICALS

Rational Exponents, Radicals, Simplifying, Operations with Radicals, Equations, Complex Numbers
What does a radical sign look like?
Here are some examples: $\sqrt{}$, $\sqrt[3]{}$, $\sqrt[4]{}$, $\sqrt[5]{}$

Square root: $\sqrt{49}$  $\sqrt[8]{81}$  $\sqrt{-16}$

Cube root: $\sqrt[3]{8}$  $\sqrt[3]{-27}$  $\sqrt[3]{1}$  $\sqrt[3]{64}$

Fourth root: $\sqrt[4]{16}$  $\sqrt[4]{(-2)^4}$  $\sqrt[4]{-16}$

Fifth root: $\sqrt[5]{32}$  $\sqrt[5]{-32}$  $\sqrt[5]{(-3)^5}$

**Even** root of a negative number is NOT real.

**Odd** root of a negative number is a negative number.

Convert rational exponents to radicals:

$8^{\frac{4}{9}}$  
$\left(5x^2y\right)^{\frac{2}{5}}$

Convert radicals to exponents. Simplify where possible.

$\sqrt{49}$  $\sqrt[3]{8}$  $\sqrt[5]{32}$

$\sqrt[4]{7^3}$  $\left(\sqrt[3]{x}\right)^5$

$\sqrt[5]{P^{20}}$  $\sqrt[3]{(5x^2y^2)^{12}}$
Radicals

Do all work on notebook paper. All steps should be shown. All work should be neat and organized.

Evaluate each of the following, if possible.

1. $\sqrt{100}$
2. $\sqrt{\frac{121}{4}}$
3. $\sqrt[3]{27}$
4. $\sqrt[3]{64}$
5. $\sqrt{-49}$
6. $\sqrt[3]{\frac{8}{27}}$
7. $\sqrt[3]{4^3}$
8. $\sqrt{13^2}$
9. $\sqrt[4]{\frac{1}{16}}$
10. $\sqrt[5]{\frac{32}{243}}$
11. $\sqrt[4]{\frac{1}{81}}$
12. $\sqrt[5]{-32}$
13. $\frac{\sqrt{100}}{\sqrt{49}}$
14. $\frac{\sqrt{27}}{\sqrt{64}}$
15. $\sqrt[8]{81}$

Convert the following expressions to radicals.

16. $7^{\frac{2}{3}}$
17. $(-5)^{\frac{1}{5}}$
18. $(4xy)^{\frac{3}{4}}$
19. $(2x)^{\frac{1}{2}}$

Convert the following radicals to expressions with rational exponents. Simplify where possible.

20. $\sqrt{7}$
21. $(\sqrt[3]{x})^8$
22. $\sqrt[3]{k^4}$
23. $\sqrt{(7x^3y)^6}$
24. $\sqrt[3]{a^{15}}$
Notes-- Rational Exponents

Evaluate each of the following, if possible.
   1. Make any negative exponents positive.
   2. Change to radicals

1. \(100^{\frac{1}{2}}\)  
2. \((-25)^{\frac{1}{2}}\)  
3. \(-25^{\frac{1}{2}}\)

4. \(-64^{\frac{2}{3}}\)  
5. \(\left(\frac{81}{49}\right)^{\frac{3}{2}}\)  
6. \(-\left(\frac{1}{32}\right)^{\frac{2}{5}}\)

7. \(25^{\frac{1}{2}}\)  
8. \((-25)^{-\frac{1}{2}}\)  
9. \((-32)^{-\frac{3}{5}}\)

10. \(8^{\frac{1}{2}} \cdot 8^{\frac{1}{6}}\)  
11. \(\frac{8^{\frac{1}{2}}}{8^{\frac{1}{6}}}\)

Simplify. All answers should have only POSITIVE exponents.

12. \(x^{\frac{1}{5}} \cdot x^{\frac{3}{4}}\)  
13. \((25x^{3})^{\frac{1}{2}} \cdot (3x^{\frac{3}{4}})\)  
14. \(\frac{-5x^{-2}y}{10x^{\frac{1}{4}}y^{\frac{2}{3}}}\)
Rational Exponents

Do all work on notebook paper. All steps should be shown. All work should be neat and organized.

Evaluate each of the following, if possible.

1. \(36^{\frac{1}{2}}\)  
2. \(\left(\frac{25}{49}\right)^{\frac{1}{2}}\)  
3. \((-9)^{\frac{1}{2}}\)  
4. \(-4^{\frac{1}{2}}\)  
5. \((-16)^{\frac{1}{4}}\)  
6. \(-16^{\frac{1}{4}}\)  
7. \(64^{\frac{1}{3}}\)  
8. \(\left(-\frac{8}{125}\right)^{\frac{1}{3}}\)  
9. \(-\left(\frac{27}{8}\right)^{\frac{1}{3}}\)  
10. \(\left(-\frac{1}{32}\right)^{\frac{1}{5}}\)  
11. \(4^{\frac{1}{2}}\)  
12. \(-8^{\frac{2}{3}}\)  
13. \(-\left(\frac{1}{16}\right)^{\frac{1}{2}}\)  
14. \(49^{\frac{1}{2}}\)  
15. \(25^{\frac{3}{2}}\)  
16. \((-49)^{-\frac{3}{2}}\)  
17. \(-100^{-\frac{1}{2}}\)  
18. \(\left(\frac{16}{81}\right)^{\frac{3}{4}}\)  
19. \(25^{\frac{5}{3}} \cdot 25^{\frac{1}{6}}\)  
20. \(\frac{27^{\frac{1}{2}}}{27^{\frac{1}{6}}}\)

Simplify each of the following. Your answers should have no NEGATIVE exponents.

21. \(x^{\frac{3}{4}} \cdot x^{\frac{1}{12}}\)  
22. \(\left(4x^3\right)^{\frac{1}{2}} \cdot \left(8x^{\frac{1}{4}}\right)\)  
23. \(\frac{2x^{\frac{2}{3}}}{3x^{\frac{1}{4}}}\)  
24. \(\frac{-6x^3y^{-\frac{1}{2}}}{4x^{\frac{1}{4}}y^2}\)  
25. \(2x^{\frac{5}{3}} \cdot 5x^{\frac{1}{6}}\)
Notes-- Simplify Radical Expressions—Part 1

Simplify the following:

\[ \sqrt{12} \quad \sqrt{98} \quad \sqrt{150} \]

\[ \sqrt[3]{40} \quad \sqrt[3]{54} \quad \sqrt[3]{240} \]

\[ \sqrt[4]{48} \quad \sqrt[4]{240} \]

\[ \sqrt{x^3 y^2} \quad \sqrt[4]{x^3 y^2} \]

\[ \sqrt[4]{x^4 y^5 z^6} \quad \sqrt[4]{x^{100} y^{52} z^{31}} \]

1. Prime factor the number.
2. For square root:
   Look for pairs.
For cube root:
   Look for 3 of a kind.
For 4th roots:
   Look for 4 of a kind. etc.
3. "Take out" the pairs, 3 of kind, etc.
Rationalize the denominator means to eliminate any radicals in the denominator.

A process to follow is:
1. **Reduce** the fraction, if possible.
2. **Simplify** the radicals
3. **Rationalize** by multiplying by "what you need".
4. **Reduce** again if necessary.

Simplify the following:

**SQUARE ROOTS:**

1. \( \frac{2}{\sqrt{3}} \)
2. \( \frac{\sqrt{50}}{\sqrt{32}} \)
3. \( \frac{3}{\sqrt{2y}} \)
4. \( \frac{\sqrt{5x}}{\sqrt{20x^2}} \)
5. \( \sqrt[3]{\frac{1}{12x^3}} \)
6. \( \sqrt[3]{\frac{49x^3}{9y^3}} \)

**CUBE ROOTS:**

7. \( \sqrt[3]{\frac{5}{2y}} \)
8. \( \sqrt[3]{\frac{5}{9y}} \)
9. \( \sqrt[3]{\frac{2y}{9x^5}} \)
Simplify Radical Expressions

Do all work on notebook paper. All steps should be shown. All work should be neat and organized.

Simplify the following radicals.

1. \(\sqrt{48}\)
2. \(\sqrt{80}\)
3. \(\sqrt[3]{16}\)
4. \(\sqrt[4]{80}\)
5. \(\sqrt[3]{128x^2}\)
6. \(\sqrt{x^2y^2}\)
7. \(\sqrt[3]{32x^7y^4}\)
8. \(\sqrt[8]{144x^5y^7z^8}\)
9. \(\sqrt[3]{81x^9y^2z^4}\)
10. \(\frac{1}{\sqrt{3}}\)
11. \(\frac{\sqrt[3]{3}}{\sqrt[3]{12}}\)
12. \(\frac{\sqrt{3}}{\sqrt[3]{5}}\)
13. \(\frac{3}{\sqrt{6}}\)
14. \(\frac{5}{\sqrt{x}}\)
15. \(\frac{5}{\sqrt{8x}}\)
16. \(\frac{\sqrt[3]{1}}{\sqrt[4]{4}}\)
17. \(\frac{\sqrt[3]{1}}{\sqrt{2}}\)
18. \(\frac{\sqrt[2]{2}}{\sqrt{9x}}\)
19. \(\frac{\sqrt[3]{3}}{\sqrt{7x}}\)
20. \(\frac{\sqrt[2]{2x}}{\sqrt[3]{3y^3}}\)
21. \(\frac{\sqrt[4]{8x^4}}{\sqrt[5]{5y^2}}\)
22. \(\frac{\sqrt[3]{7}}{\sqrt{8x}}\)
23. \(\frac{\sqrt[3]{16}}{\sqrt[3]{3x^2}}\)
24. \(\frac{\sqrt[3]{3y^2}}{\sqrt[3]{5x^4}}\)
25. \(\frac{\sqrt[3]{49x^3}}{\sqrt[3]{9y^3}}\)
Review of collecting like terms:

\[ 2x + 5x \]

\[ 2x^2 + 5x \]

Perform the indicated operations:

1. \[ 4\sqrt{2} + 3\sqrt{2} - \sqrt{2} \]

2. \[ -\sqrt{40} - \sqrt{90} - \sqrt{160} \]

3. \[ x^2\sqrt{18y} - 2x\sqrt{2x^2y} + 4y\sqrt{12x} - \sqrt{3xy^2} \]

4. \[ \sqrt[3]{250xy^2} - \sqrt[3]{54xy^2} \]
Perform the indicated operations:

1. \( \frac{4}{\sqrt{3}} - 7\sqrt{3} \)

2. \( \frac{5}{\sqrt{6}} + \sqrt{24} \)

3. \( \frac{2}{\sqrt{3}} - \sqrt{72} \)

4. \( \sqrt[3]{\frac{2}{3}} + 5\sqrt{6} - \sqrt[2]{\frac{3}{2}} \)

1. Simplify radicals.
2. Rationalize all denominators.
3. Find the LCD
4. Re-write all terms with LCD.
5. Combine like terms.
Adding and Subtracting Radical Expressions

Do all work on notebook paper. All steps should be shown. All work should be neat and organized.

Perform the indicated operations.

1. $3\sqrt{7} - 5\sqrt{7} - 8\sqrt{7}$
2. $2\sqrt{15x} - 7\sqrt{15x} + 5\sqrt{15x}$
3. $x\sqrt{10y} - 8x\sqrt{10y}$
4. $5\sqrt{12} - 2\sqrt{27} + 3\sqrt{3}$
5. $5\sqrt{8} - \sqrt{18} + 2\sqrt{32}$
6. $\frac{3}{32} + \frac{3}{108}$
7. $\sqrt{27x} - 2\sqrt{12x} + 2\sqrt{48x}$
8. $x\sqrt{18x} + 5\sqrt{2x^3} + 2x\sqrt{8x}$
9. $\sqrt{27x} + 2\sqrt{12x} - \sqrt{150y} - 4\sqrt{24y}$
10. $x\sqrt{20x^2} - x\sqrt{45} + 2\sqrt{500}$
11. $\frac{1}{\sqrt{2}} + 5\sqrt{2}$
12. $\frac{3}{\sqrt{5}} + \sqrt{80}$
13. $\frac{1}{\sqrt{2}} - \frac{3}{\sqrt{108}}$
14. $\sqrt{3} + \frac{5}{\sqrt{3}} + \frac{2\sqrt{27}}{3}$
15. $\sqrt{2} - \frac{4}{\sqrt{2}} - \frac{\sqrt{50}}{2}$
Multiply Radicals

Multiplying a monomial by a monomial:
Multiply the "outsides"
Multiply the "insides"
Simplify, if possible.

\[(2\sqrt{3})(4\sqrt{15})\] \[(2\sqrt{3}y)(5\sqrt{2}x)\]

Multiply a square root by the SAME square root:
\[(\sqrt{3})(\sqrt{3})\] \[(\sqrt{7}y)(\sqrt{7}y)\] \[(\sqrt{5} - y)(\sqrt{5} - y)\]

Perform the indicated operations and simplify your answers:

1. \[3\sqrt{5y}\left(\sqrt{5}y + \sqrt{x} - 2\sqrt{3}\right)\]
2. \[(4\sqrt{2} - 2\sqrt{3})(5\sqrt{3} - 8\sqrt{6})\]
3. \[(3\sqrt{x} - 2\sqrt{y})^2\]
4. \[(5 + \sqrt{x} + 3)^2\]
5. \[(2 - 3\sqrt{5x} - 1)^2\]
Divide Radicals

Review: 

\[(x + 5)(x - 5) \quad (\sqrt{x} + 5)(\sqrt{x} - 5) \quad (\sqrt{3} + \sqrt{5})(\sqrt{3} - \sqrt{5})\]

The "conjugate" of \(\sqrt{x} - 5\) is \(\sqrt{x} + 5\)  

The "conjugate" of \(\sqrt{3} + \sqrt{5}\) is \(\sqrt{3} - \sqrt{5}\)

As you can see, the conjugate is found by changing the middle sign.  
When you multiply conjugates, you just need to square each term and then subtract.  
We use the conjugate to rationalize the binomial denominators.

Rationalize the denominator of the following.

1. \(\frac{1}{\sqrt{5} - 1}\)  
2. \(\frac{\sqrt{3}}{\sqrt{6} + \sqrt{2}}\)

3. \(\frac{\sqrt{8} - \sqrt{2}}{\sqrt{6} - \sqrt{8}}\)  
4. \(\frac{\sqrt{2x} - \sqrt{y}}{\sqrt{3x} + \sqrt{5y}}\)
Operations with Radical Expressions

Do all work on notebook paper. All steps should be shown. All work should be neat and organized.

Perform the indicated multiplications and simplify your answers.

1. \( \sqrt{3} \left( 2 + \sqrt{5} \right) \)
2. \( \sqrt{x} \left( 3 + \sqrt{x} \right) \)
3. \( (2 + \sqrt{7}) \left( \sqrt{8} - 3 \right) \)
4. \( (\sqrt{5} + \sqrt{7}) \left( \sqrt{10} + \sqrt{2} \right) \)
5. \( (2\sqrt{3} + 4\sqrt{5}) \left( 3\sqrt{2} - 6\sqrt{7} \right) \)
6. \( (2\sqrt{x} + 3\sqrt{y}) \left( 4\sqrt{x} + 5\sqrt{y} \right) \)

Rationalize the denominator of the following radicals.

13. \( \frac{1}{\sqrt{3} + 2} \)
14. \( \frac{2}{\sqrt{3} + \sqrt{5}} \)
15. \( \frac{x}{\sqrt{x} + \sqrt{y}} \)
16. \( \frac{2 + \sqrt{3}}{2 - \sqrt{3}} \)
17. \( \frac{\sqrt{x} + 5}{\sqrt{x} - 4} \)
18. \( \frac{\sqrt{5} + \sqrt{2}}{\sqrt{6} - \sqrt{3}} \)
19. \( \frac{\sqrt{x} + \sqrt{y}}{\sqrt{x} - \sqrt{y}} \)
20. \( \frac{5\sqrt{3} + 7}{2\sqrt{3} - 4} \)
Notes-- Radical Equations

Process:
1. Isolate the radical.
2. Get rid of the radical by raising both sides to the appropriate power.
   \[(\sqrt{x})^2 = x\]  \[\left(\sqrt[3]{x}\right)^3 = x\]  \[\left(\sqrt[4]{x}\right)^4 = x\]
3. Solve the resulting equation.
4. Check for extraneous solutions.

1. \[\sqrt{4x+1} - 5 = 0\]  
2. \[\sqrt[3]{x^2 + 4} - 1 = 4\]

3. \[\sqrt{x^2 + 16} + 6 = 1\]  
4. \[\sqrt[4]{x^2 + x} - 4 = 2\]

5. \[2x = \sqrt{4x + 15}\]  
6. \[\sqrt{3x + 4} - 2 = x\]
Remember that a fractional exponent can be written in radical form.

\[ x^{\frac{1}{2}} = \sqrt{x^3} \text{ or } (\sqrt{x})^3 \]
\[ x^{\frac{2}{5}} = \sqrt[5]{x^2} \text{ or } (\sqrt[5]{x})^2 \]

If you encounter an equation that has a variable raised to a fractional exponent, you solve it just like a radical equation.

**Get rid of the radical by raising both sides to the appropriate power.**

\[
\left(x^{\frac{3}{2}}\right)^{\frac{2}{3}} = x \quad \left(x^{\frac{2}{5}}\right)^{\frac{5}{2}} = x
\]

7. \( (x^2 + 6x - 7)^{\frac{3}{2}} = 27 \)

8. \( (x - 2)^{\frac{2}{3}} = 9 \)
Radical Equations

Do all work on notebook paper. All steps should be shown. All work should be neat and organized.

Find the solutions(s) of the following radical equations.

1. \( \sqrt{2x+1} = 3 \)  
11. \( \sqrt{2x+5} - x = 3 \)
2. \( \sqrt{x^2 + 7} - 4 = 0 \)  
12. \( \sqrt{x+8} - x = -4 \)
3. \( \sqrt{3x-7} + 2 = 0 \)  
13. \( \sqrt{3x+5} + 1 = 3x \)
4. \( \sqrt[3]{3x+1} = 4 \)  
14. \( \sqrt{3-x} - x = 3 \)
5. \( \sqrt[3]{3x^2 - 10x + 2} = 0 \)  
15. \( \sqrt{x-4} + x = 6 \)
6. \( \sqrt[4]{3x+5} = -3 \)  
16. \( (x^2 + 4x - 5)^{\frac{3}{2}} = 64 \)
7. \( \sqrt{3x+40} = x \)  
17. \( (2x^2 - 5x + 6)^{\frac{3}{2}} = 8 \)
8. \( \sqrt{2x^2 + 2x - 3} = x \)  
18. \( (x-7)^{\frac{3}{2}} = 4 \)
9. \( \sqrt{4x^2 - x - 1} + 1 = 2x \)  
19. \( (2x+5)^{\frac{3}{2}} = 9 \)
10. \( \sqrt{x^2 - 8x + 26} + 5 = x \)  
20. \( (x-1)^{\frac{3}{2}} = 4 \)
What happens when we want to solve the equation: \( x^2 = -4 \)?

In order to solve this equation, we must introduce \( \sqrt{-1} \) and the set of imaginary numbers.

We will represent \( \sqrt{-1} \) with \( i \).

\[
\left( \sqrt{-1} \right)^2 = i^2
\]

\[
(-1) = i^2
\]

Therefore, \( -1 = i^2 \star \)

Any square root of a negative number can be written in terms of \( i \).

\[
\sqrt{-2} = \]

\[
\sqrt{-9} = \]

A complex number has a real part and an imaginary part.

\[
3 + 5i
\]

We can add, subtract, multiply, and divide complex numbers.

Perform the indicated operations:

1. \((-4 + 7i) + (3 + 2i)\)

2. \((3 - 5i) - (7 + 4i)\)

3. \((2 + 3i)(4 + 5i)\)

4. \((4 - i)^2\)  \( \star \) Replace \( i^2 \) with \(-1\)
Rationalize the denominators. (Divide)

Review: \( \frac{3}{5\sqrt{2}} \)

Review: \( \frac{3}{4+\sqrt{5}} \)

The "complex conjugate" of \( 3+5i \) is \( 3-5i \)

Multiply: \((3+5i)(3-5i)\)

5. \( \frac{3-5i}{2i} \)

6. \( \frac{2i}{3-5i} \)

7. \( \frac{3+2i}{-5+4i} \)
Complex Numbers

Do all work on notebook paper. All steps should be shown. All work should be neat and organized.

Perform the indicated operations.

1. \((2 + 3i) + (5 - 6i)\)
2. \((-5 - 2i) - (3 + i)\)
3. \(7(3 + 9i)\)
4. \(2i(4 - 7i)\)
5. \((6 + i)(3 - 4i)\)
6. \((5 - 3i)(2 - 4i)\)
7. \((2 + 5i)(2 - 5i)\)
8. \((\sqrt{3} + i)(\sqrt{3} - i)\)
9. \((2 + 3i)^2\)
10. \((\sqrt{3} + i)^2\)

Rationalize the denominators.

11. \(\frac{5}{3i}\)
12. \(\frac{5 + 3i}{i}\)
13. \(\frac{4 + i}{2i}\)
14. \(\frac{1}{4 + 2i}\)
15. \(\frac{3}{2 - 3i}\)
16. \(\frac{2i}{1 + 3i}\)
17. \(\frac{-4i}{-3 - 5i}\)
18. \(\frac{3 - 4i}{3 + i}\)
19. \(\frac{9 - 17i}{2 - i}\)
20. \(\frac{-2 - 5i}{1 - 4i}\)
QUADRATICS

Extraction of Roots, Completing the Square, Quadratic Formula, Applications
This is another method to solve quadratic equations. If the quadratic cannot be factored we have to have something else that will allow us to solve the equation. There are 2 such methods—completing the square and the quadratic formula. The quadratic formula is derived from completing the square on the general equation: 

$$ax^2 + bx + c = 0$$

You MUST memorize the formula: 

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

**Process:**
1. Write the equation in standard form: 
   $$ax^2 + bx + c = 0$$
2. Identify $a$, $b$, and $c$. 
3. Substitute numbers into formula. 
4. Carefully do the arithmetic under the square root sign. 
5. If possible, simplify the radical. 
6. If possible, reduce the fraction.

1. $x^2 - 4x - 1 = 0$ 
2. $9x^2 - 18x + 7 = 0$

3. $x^2 + 9x + 11 = 3x - 2$ 
4. $x(x + 2) = 6x - 11$

5. $(3x - 2)(x + 4) = -7$
Quadratic Formula

Do all work on notebook paper. All steps should be shown. All work should be neat and organized.

Use the quadratic formula to find the solutions of the following quadratic equations.

1. \( x^2 + 9x - 9 = 0 \)
2. \( 9x^2 + 2x - 2 = 0 \)
3. \( 8x^2 + 10x + 1 = 0 \)
4. \( 6x^2 + 2x + 5 = 0 \)
5. \( x^2 + 6x - 8 = 0 \)
6. \( 8x^2 - x + 2 = 0 \)
7. \( 4x^2 - 6x - 9 = 0 \)
8. \( 2x^2 + 8x + 7 = 0 \)
9. \( 4x^2 + 4x + 3 = 0 \)
10. \( 8x^2 - 4x - 5 = 0 \)
11. \( x(2x + 3) = 7 \)
12. \( 2x(x + 3) = -1 \)
13. \( (x + 5)(3x - 1) = 4 \)
14. \( \frac{1}{2} - \frac{1}{x} - \frac{1}{x^2} = 0 \)
Another method of solving quadratic equations is the **extraction of roots**. This is very convenient if the equation has a squared term that can be isolated. Some examples are:

\[
x^2 = 81 \\
9x^2 - 23 = 0 \\
(x - 1)^2 = 25 \\
(2x - 10)^2 = 12
\]

**Process:**
1. Isolate the squared term.
2. Take the square root of both sides of the equation.
   Don’t forget the “±” sign.
3. Simplify all radicals. Rationalize all denominators.
4. Solve the equation.

1. \(x^2 = 81\)
2. \(9x^2 - 23 = 0\)
3. \((x - 1)^2 = 25\)
4. \((2x - 10)^2 = 12\)
5. \((5x + 3)^2 = -28\)
Completing the Square

This is another method to solve quadratic equations. If the quadratic cannot be factored we have to have something else that will allow us to solve the equation. There are 2 such methods—completing the square and the quadratic formula. Completing the Square is also used for other applications.

Process:
1. Write the equation in standard form: \( ax^2 + bx + c = 0 \)
2. Move \( c \) to the left hand side of the equation.
   \[ x^2 + bx + ____ = -c + ____ \]
3. If \( a \) is NOT = 1, divide all terms by \( a \). Reduce any fractions.
4. Take \( \frac{1}{2} \) of the coefficient of \( x \).
5. Square this and add to both sides of the equation.
6. Re-write left hand side as a squared binomial.
7. Solve the equation by the extraction of roots method.

1. \( x^2 + 8x - 11 = 0 \)  
2. \( x^2 - 6x + 18 = 0 \)  
3. \( x^2 + 3x - 13 = 0 \)

4. \( 2x^2 - 2x + 10 = 0 \)  
5. \( 3x^2 + 5x + 7 = 0 \)
Completing the Square

Do all work on notebook paper. All steps should be shown. All work should be neat and organized.

Use the Extraction of Roots method to find the solutions of the following quadratic equations.

1. \( x^2 = 25 \)
2. \( x^2 = 80 \)
3. \( x^2 = -121 \)
4. \( x^2 - 64 = 0 \)
5. \( 4x^2 + 81 = 0 \)
6. \( 9x^2 - 49 = 0 \)
7. \( 5x^2 - 8 = 0 \)
8. \( 7x^2 - 2 = 0 \)
9. \( 2x^2 - 3 = 8 \)
10. \( (2x - 7)^2 = 6 \)
11. \( (4x - 5)^2 = 9 \)
12. \( (x - 1)^2 = -75 \)
13. \( (3x + 5)^2 = 27 \)
14. \( (5x + 1)^2 = 50 \)

Find the solutions of the following quadratic equations by completing the square.

15. \( x^2 + 4x - 1 = 0 \)
16. \( x^2 - 2x - 7 = 0 \)
17. \( x^2 + 4x + 29 = 0 \)
18. \( x^2 - 2x + 37 = 0 \)
19. \( x^2 + 5x - 2 = 0 \)
20. \( 4x^2 + 4x - 2 = 0 \)
21. \( 2x^2 - 5x - 4 = 0 \)
22. \( 4x^2 - 4x + 17 = 0 \)
23. \( 4x^2 + 12x + 7 = 0 \)
**Applications—Quadratic Equations**

**Consecutive integers:** Remember that integers are only negative and positive “whole numbers”. They do not include any decimals or fractions.
- Two consecutive integers: \( x, x+1 \)
- Two consecutive **odd** integers: \( x, x+2 \)
- Two consecutive **even** integers: \( x, x+2 \)

**Reciprocals:** If the number is \( x \), then its reciprocal would be \( \frac{1}{x} \).

*Define the variable. Write an equation. Solve.*

<table>
<thead>
<tr>
<th>1. Find two consecutive positive integers whose product is 132.</th>
<th>2. Find two consecutive odd integers whose product is 35.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>3. The sum of the squares of two even consecutive integers is 340. Find the integers.</th>
<th>4. The sum of a number and its reciprocal is 6. What is the number?</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>5. Three times the square of a number is 6 more than twice the number. What is the number?</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
</tbody>
</table>
Area:

Rectangle: Area = base \times height or Area = length \times width.

Triangle: Area = \frac{1}{2} \times base \times height or 2 \times Area = base \times height

6. The area of a rectangle is 35 sq. ft. The length is 1 ft. less than twice the width. What are the dimensions of the rectangle?

7. The height of a triangle is 2 in. more than three times the base. Find the base and the height if the area of the triangle is 5 sq. in.

Work:

\frac{1}{time \ alone} + \frac{1}{time \ alone} = \frac{1}{time \ together}

8. One pipe can fill a reservoir 2 hours faster than another pipe can. Together they fill the reservoir in 5 hours. How long does it take each pipe to fill the reservoir?
Applications-Quadratics Equations

Do all work on notebook paper. All steps should be shown. All work should be neat and organized.

Find the solutions to the following problems.

1. Find two consecutive integers whose product is 72.
2. Find two consecutive odd integers whose product is 63.
3. The sum of a number and its reciprocal is 4. What is the number?
4. Find two consecutive even integers whose product is 48.
5. The sum of the squares of two consecutive integers is 25. Find the integers.
6. The sum of the squares of two consecutive integers is 61. Find the integers.
7. One number is 1 more than twice another number. The product of the two numbers is 21. What are the numbers?
8. The square of a number minus 2 times the number is 14. What is the number?
9. The area of a rectangle is 42 sq. ft. The length is 2 ft. more than 4 times the width. What are the dimensions of the rectangle?
10. The base of a triangle is 1 inch more than twice the height. Find the base and the height if the area of the triangle is 14 sq. in.?
11. The base of a triangle is 1 inch less than twice the height. Find the base and the height if the area of the triangle is 6 sq. in.?**
12. One pipe can fill a reservoir 1 hour faster than another pipe. Together they fill the reservoir in 4 hours. How long does it take for each pipe to fill the reservoir?**
13. Gary can process his job 2 days faster than his co-worker, Bob. Working together, they can process the job in 6 days. Working alone, how long does it take each of the men to process the job?**

**Determine a decimal approximation.
1. \( A(4,0) \) \( B(1,6) \) \( C(0,4) \) \( D(0,0) \) \( E(-3,2) \) \( F(-6,0) \) \( G(-4,-5) \) \( H(0,-2) \) \( I(7,-6) \)

2. \( y = 2x - 3 \)

3. \( x + 2y = 4 \)

4. \( \frac{1}{2}x + 2 = y \)

5. \( 2x + 3y = 0 \)

6. \( y = x^2 - 5 \)

7. \( y = 9 - x^2 \)

8. \( x = 1 - y^2 \)

9. \( y = \sqrt{x - 4} \)

10. \( y = \sqrt{x + 2} \)
1. \( V = \left\{ (-5,17),(0,7),(2,3),(1,5),\left(\frac{3}{2},4\right) \right\} \) Domain= \( \{-5,0,2,1,\frac{3}{2}\} \) Range= \( \{17,7,3,5,4\} \)

2. \( Z = \{(-2,5),(0,3),(2,5)\} \) Domain= \( \{-2,0,2\} \) Range= \( \{3,5\} \)

3. \( X = \left\{ (1,1),(3,\sqrt{5}),(5,3),\left(\frac{1}{2},0\right) \right\} \) Domain= \( \{1,3,5,\frac{1}{2}\} \) Range= \( \{1,\sqrt{5},3,0\} \)

4. Domain= \( (-\infty,\infty) \) Range= \( (-\infty,\infty) \); Yes, it is a function

5. Domain= \( [-3,3] \) Range= \( [0,3] \); Yes, it is a function

6. Domain= \( (-\infty,\infty) \) Range= \( [0,\infty) \); Yes, it is a function

7. Domain= \( (-\infty,\infty) \) Range= \( (-\infty,2] \); Yes, it is a function

8. Domain= \( (-\infty,1] \) Range= \( (-\infty,0] \); Yes, it is a function

9. Domain= \( [-2,2] \) Range= \( [0,4] \); Not a function

10. Domain= \( [4,\infty) \) Range= \( (-\infty,\infty) \); Not a function

11. Domain= \( [-6,4] \) Range= \( [-2,4]\); Yes, it is a function

12. Domain= \( (-\infty,\infty) \) Range= \( [4]\); Yes, it is a function

13. Domain= \( [3]\) Range= \( (-\infty,\infty) \); Not a function

14. Domain= \( [-2,\infty) \) Range= \( (-\infty,5]\); Yes, it is a function

15. Domain= \( (-\infty,-2] \) Range= \( (-3,\infty) \); Yes, it is a function

16. Domain= \( [-5,5] \) Range= \( [-3,3]\); Not a function

17. Domain= \( (-\infty,\infty) \) Range= \( (-\infty,3]\); Yes, it is a function

18. Domain= \( (-\infty,1] \) Range= \( (-\infty,\infty) \); Not a function

19. Domain= \( [-2,\infty) \) Range= \( (-\infty,-2]\cup(0,4]\); Yes, it is a function
### Function Notation and Operations of Functions-Answers Page 21

1. \( f(-2) = -7, f(-1) = -5, f(0) = -3, f(1) = -1, f(a) = 2a - 3, f(a + h) = 2a + 2h - 3 \)
2. \( f(-2) = 5, f(-1) = 4, f(0) = 3, f(1) = 2, f(a) = 3 - a, f(a + h) = 3 - a - h \)
3. \( f(-2) = 2, f(-1) = -1, f(0) = -2, f(1) = -1, f(a) = a^2 - 2, f(a + h) = a^2 + 2ah + h^2 - 2 \)
4. \( f(-2) = 1, f(-1) = 4, f(0) = 5, f(1) = 4, f(a) = 5 - a^2, f(a + h) = 5 - a^2 - 2ah - h^2 \)
5. \( f(-2) = 8, f(-1) = 2, f(0) = 0, f(1) = 2, f(a) = 2a^2, f(a + h) = 2a^2 + 4ah + 2h^2 \)

\[
(f + g)(x) = 2x^2 + 6x - 3, (f - g)(x) = 2x^2 + 4x + 1
\]
6. \( (f \cdot g)(x) = 2x^3 + x^2 - 11x + 2, \left(\frac{f}{g}\right)(x) = \frac{2x^2 + 5x - 1}{x - 2} \)

\[
(f + g)(x) = 2x^2 - 4, (f - g)(x) = 14
\]
7. \( (f \cdot g)(x) = x^4 - 4x^2 - 45, \left(\frac{f}{g}\right)(x) = \frac{x^2 + 5}{x^2 - 9} \)

\[
(f + g)(x) = 3x - 8, (f - g)(x) = x + 14
\]
8. \( (f \cdot g)(x) = 2x^2 - 19x - 33, \left(\frac{f}{g}\right)(x) = \frac{2x + 3}{x - 11} \)

\[
(f + g)(x) = x^3 + 5x - 4, (f - g)(x) = x^3 + x - 6
\]
9. \( (f \cdot g)(x) = 2x^4 + x^3 + 6x^2 - 7x - 5, \left(\frac{f}{g}\right)(x) = \frac{x^3 + 3x - 5}{2x + 1} \)

10. \(-7\) 17. \(0\) 24. \(3\)
11. \(5\) 18. \(2x^2 + 5x + 3\) 25. \(3a^2 + 5a - 1\)
12. \(-15\) 19. \(\frac{x^2 - 4}{2x + 1}\) 26. \(3a^2 + 6ah + 3h^2 + 5a + 5h - 1\)
13. \(-343\) 20. \(12x^2 + 10x - 1\) 27. \(6a + 3h + 5\)
14. \(41\) 21. \(6x + 1\) 22. \(3a - 1\)
15. \(\frac{25}{1681}\) 23. \(3a + 3h - 1\)
16. \(3\)
1. \( y = 2x - 6; m = 2; \text{y-intercept} = -6 \)

2. \( y = -\frac{3}{4}x + 2; m = -\frac{3}{4}; \text{y-intercept} = 2 \)

3. \( y = -\frac{1}{3}x; m = -\frac{1}{3}; \text{y-intercept} = 0 \)

4. \( y = -\frac{2}{3}x - \frac{8}{3}; m = -\frac{2}{3}; \text{y-intercept} = -\frac{8}{3} \)

5. \( y = 3; m = 0; \text{y-intercept} = 3 \)

6. \( y = -\frac{9}{4}x + 6; m = -\frac{9}{4}; \text{y-intercept} = 6 \)

7. \( \text{x-intercept} = 2; \text{y-intercept} = \frac{3}{2} \)

8. \( \text{x-intercept} = \frac{5}{2}; \text{y-intercept} = -2 \)

9. \( \text{x-intercept} = 0; \text{y-intercept} = 0 \)

10. \( \text{x-intercept} = -\frac{5}{2}; \text{y-intercept} = 5 \)

11. \( \text{x-intercept} = 15; \text{y-intercept} = -\frac{9}{2} \)

12. \( m = \frac{1}{2} \)

13. \( m = -\frac{6}{7} \)

14. \( m = 3 \)

15. \( m = 0 \)

16. \( m = \text{undefined} \)
17. \( m = 3; \)  
\( y \)-intercept = -2

18. \( m = -\frac{1}{2}; \)  
\( y \)-intercept = 2

19. \( m = \frac{3}{4}; \)  
\( y \)-intercept = 2

20. \( m = -\frac{3}{5}; \)  
\( y \)-intercept = 0

21. \( m = -2; \)  
\( y \)-intercept = \( \frac{5}{2} \)

22. \( m = 0; \)  
\( y \)-intercept = 4

23. \( m \) undefined;  
\( y \)-intercept \textit{NONE}

24. \( m = 0; \)  
\( y \)-intercept = -2

25. \( m \) undefined;  
\( y \)-intercept \textit{NONE}
Equations of Lines-Answers Page 35

1. \( y = \frac{5x}{2} + \frac{1}{2} \)
2. \( y = x - 9 \)
3. \( y = \frac{1}{2}x - 3 \)
4. \( y = 11 \)
5. \( y = \frac{2}{5} \)
6. \( y = 3x + 1 \)
7. \( y = \frac{9}{2}x - \frac{11}{2} \)
8. \( y = -\frac{2}{3}x + \frac{19}{3} \)
9. \( y = -12 \)
10. \( x = 3 \)
11. \( y = -x + 2 \)
12. \( y = -\frac{3}{4}x \)
13. \( y = -\frac{3}{2}x \)
14. \( y = -4 \)
15. \( x = 5 \)

Parallel and Perpendicular Lines-Answers Page 39

1. \( y = 3x - 5 \)
2. \( y = -\frac{5}{4}x + \frac{59}{4} \)
3. \( y = \frac{3}{7}x + \frac{41}{7} \)
4. \( y = -7 \)
5. \( x = -8 \)
6. \( y = -\frac{1}{3}x + \frac{5}{3} \)
7. \( y = -\frac{5}{3}x + 16 \)
8. \( y = \frac{3}{4}x + \frac{13}{2} \)
9. \( x = 3 \)
10. \( y = -9 \)
1. \( y - 2x < 6 \)
   - \( y < 2x + 6 \)
   - \( m = 2 \)
   - \( b = 6 \)

2. \( 3x + 2y \geq 12 \)
   - \( y \geq -\frac{3}{2}x + 6 \)
   - \( m = -\frac{3}{2} \)
   - \( b = 6 \)

3. \( y \leq \frac{1}{3}x - 1 \)
   - \( y \leq \frac{1}{3}x - 1 \)
   - \( m = \frac{1}{3} \)
   - \( b = -1 \)

4. \( y > 4x - 5 \)
   - \( y > 4x - 5 \)
   - \( m = 4 \)
   - \( b = -5 \)

5. \( 4x \leq 2y - 6 \)
   - \( y \geq 2x + 3 \)
   - \( m = 2 \)
   - \( b = 3 \)

6. \( 7 < 3x - y \)
   - \( y < 3x - 7 \)
   - \( m = 3 \)
   - \( b = -7 \)
<table>
<thead>
<tr>
<th></th>
<th>Equation</th>
<th>Slope (m)</th>
<th>y-intercept (b)</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>$5x &gt; -3y + 9$</td>
<td>$y &gt; -\frac{5}{3}x + 3$</td>
<td>$m = -\frac{5}{3}$, $b = 3$</td>
</tr>
<tr>
<td>8</td>
<td>$-2y \leq 7x - 8$</td>
<td>$y \geq -\frac{7}{2}x + 4$</td>
<td>$m = -\frac{7}{2}$, $b = 4$</td>
</tr>
<tr>
<td>9</td>
<td>$2x &gt; 3y$</td>
<td>$y &lt; \frac{2}{3}x$</td>
<td>$m = \frac{2}{3}$, $b = 0$</td>
</tr>
<tr>
<td>10</td>
<td>$x + 4y \geq 0$</td>
<td>$y \geq -\frac{1}{4}x$</td>
<td>$m = -\frac{1}{4}$, $b = 0$</td>
</tr>
<tr>
<td>11</td>
<td>$y &lt; 3$</td>
<td>$y &lt; 3$</td>
<td>$m = 0$, $b = 3$</td>
</tr>
<tr>
<td>12</td>
<td>$x \geq -5$</td>
<td>$m = \text{undefined}$</td>
<td>$b = \text{NONE}$</td>
</tr>
</tbody>
</table>
Linear Systems in Two Variables-Answers Page 49

1. \((-1, -3)\)  
2. \((-1, 4)\)  
3. \((2, 3)\)  
4. \((0, 4)\)  
6. Same line. Dependent.  
7. \((-5, -3)\)  
8. Same line. Dependent.  
9. \((6, -1)\)  
10. \(\left(-\frac{3}{4}, 0\right)\)  
11. \((-2, -4)\)  
13. \((-2, 4)\)

Applications-System of Equations-Answers Page 53

1. \(\{4, 21\}\)  
2. \(\{8, -2\}\)  
3. \(\{4, -6\}\)  
4. 16 dimes and 24 nickels  
5. 12—20¢ stamps and 9—32¢ stamps.  
6. 12 Adult tickets and 25 children’s tickets  
7. 14¢ pencils and 6¢ erasers  
8. $9 football and $14 basketballs
Absolute Value Equations-Answers-Page 59

1. \{8, -8\}
2. \{6, -6\}
3. \{4, -3\}
4. \(\left\{\frac{7}{2}, -2\right\}\)
5. \(\emptyset\)
6. \(\left\{-\frac{8}{3}, 8\right\}\)
7. \(\left\{-\frac{5}{2}\right\}\)

Absolute Value Inequalities-Answers-Page 63

1. \((-3, 3)\)
2. \((-\infty, -2] \cup [2, \infty)\)
3. \([-4, 4]\)
4. \((-\infty, -14) \cup (14, \infty)\)
5. \(\emptyset\)
6. \((-\infty, \infty)\)
7. \(4, 6\)
8. \((-\infty, 1) \cup (4, \infty)\)
9. \((-5, 3)\)
10. \((-\infty, -4] \cup [3, \infty)\)
11. \(\left(\frac{5}{4}, \frac{9}{4}\right)\)
12. \(\emptyset\)
13. \((-\infty, \infty)\)
14. \((-\infty, -4) \cup (6, \infty)\)
15. \([0, 2]\)
GCF and Grouping-Answers Page 69

1. $5(2x^2 - 7)$
2. $2x(4x^2 + 9y + 1)$
3. $-2x^2(3x^3 + 9x + 5)$ or $2x^2(-3x^3 - 9x - 5)$
4. $(x-2y)(2x-3y)$
5. $(2x+1)(x-3)$
6. $2yz(1-5x)(4y-3z^2)$
7. $(y-3)(3x+z)$
8. $(x+y)(2a+3b)$
9. $2(3f-g)(c+d)$
10. $(3x-4)(5y-x)$ or $(4-3x)(x-5y)$
11. $xy(3-5y)(x-4y)$
12. $(3y+7)(y^2+6)$
13. $4x(2x^2 + 3)(x^3 + 5)$
14. $3(2y^2 - x^3)(9y-2x)$

Factoring Binomials-Answers Page 73

1. $(x+3)(x-3)$
2. $(6x+7y)(6x-7y)$
3. $3xy(3x+2y)(3x-2y)$
4. $3(x^2 + 16)$
5. $(9x^2 + 4y^2)(3x+2y)(3x-2y)$
6. $(x+4)(x-10)$
7. $(2x+3y+3z)(2x+3y-3z)$
8. $(x-4)(x^2 + 4x + 16)$
9. $(2x+1)(4x^2 - 2x + 1)$
10. $(4x-3y)(16x^2 + 12xy + 9y^2)$
11. $2(3x+4y)(9x^2 - 12xy + 16y^2)$
12. $3(5x-1)(25x^2 + 5x + 1)$
13. $(2x-3)(x+1)(x-1)$
14. $(x-6)(x-2)(x^2 + 2x + 4)$
15. $(5x+2)(x+3)(x-3)$
16. $(x+y)(x-y)(3x+4y)$
17. $(3x-5)(x-1)(x^2 + x + 1)$
18. $(4x-3)(x+2)(x^2 - 2x + 4)$
19. $(2x+1)(2x-1)(x-2)(x^2 + 2x + 4)$
20. $7x(x+2y)(x-2y)(x+1)(x-1)$
Factoring Trinomials-Answers Page 77

1. \((x-7)(x-2)\)  
2. \((x-8)(x-1)\)  
3. PRIME  
4. \((x-3)(x-3)\) or \((x-3)^2\)  
5. \(3(x+7)(x+1)\)  
6. \(-2x(x+4)(x+4)\)  
7. \((x+5)(x+y)\)  
8. \((3x+1)(2x-5)\)  
9. \((4x-1)(3x+7)\)  
10. \((3x-4)(3x-1)\)

Quadratic Equations-Answers Page 81

1. \((-2,-4)\)  
2. \((-8,2)\)  
3. \(0,\frac{1}{3}\)  
4. \(-\frac{3}{2},\frac{3}{2}\)  
5. \(-\frac{1}{2},\frac{4}{2}\)  
6. \(-\frac{1}{6},\frac{7}{6}\)  
7. \(\left\{\frac{9}{4},\frac{1}{3}\right\}\)  
8. \(\{-7,2\}\)  
9. \(\left\{-\frac{1}{4},\frac{5}{2}\right\}\)  
10. \(\{-4\}\)  
11. \(\left\{-\frac{7}{2},\frac{1}{3}\right\}\)  
12. \(\{-5,2\}\)  
13. \(\{2,4\}\)
### Integer Exponents-Answers Page 87

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<tr>
<td>1.</td>
<td>32</td>
<td>9.</td>
<td>$\frac{-1}{81}$</td>
<td>15.</td>
<td>$\frac{73}{8}$ or $9\frac{1}{8}$</td>
<td>21.</td>
<td>$4x^{12}y^2$</td>
<td>27.</td>
<td>$\frac{2}{x^2y^2}$</td>
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<td>2.</td>
<td>$\frac{-1}{7}$</td>
<td>10.</td>
<td>$\frac{-1}{27}$</td>
<td>16.</td>
<td>$\frac{31}{5}$ or $6\frac{1}{5}$</td>
<td>22.</td>
<td>$\frac{1}{48y^8}$</td>
<td>28.</td>
<td>$\frac{16x^{12}}{25y^{30}}$</td>
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<td>3.</td>
<td>16</td>
<td>11.</td>
<td>$\frac{-1}{27}$</td>
<td>17.</td>
<td>$\frac{8}{27}$</td>
<td>23.</td>
<td>1</td>
<td>29.</td>
<td>$\frac{-432y^{33}}{x^4}$</td>
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<tr>
<td>4.</td>
<td>$-64$</td>
<td>12.</td>
<td>$\frac{3}{8}$</td>
<td>18.</td>
<td>$\frac{8}{5}$</td>
<td>24.</td>
<td>$\frac{8x^{12}y^6}{125}$</td>
<td>30.</td>
<td>$\frac{64x^{10}}{y^4}$</td>
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<tr>
<td>5.</td>
<td>64</td>
<td>13.</td>
<td>$\frac{19}{48}$</td>
<td>19.</td>
<td>$\frac{9}{16}$</td>
<td>25.</td>
<td>$\frac{125y^9}{27x^{15}}$</td>
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<tr>
<td>6.</td>
<td>$-8$</td>
<td>14.</td>
<td>$\frac{1}{2}$</td>
<td>20.</td>
<td>$\frac{3}{25}$</td>
<td>26.</td>
<td>$\frac{4x^9}{5y^7}$</td>
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### Reducing Rational Expressions-Answers Page 91

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<tbody>
<tr>
<td>1.</td>
<td>$\frac{4x^3}{3y}$</td>
<td>4.</td>
<td>$\frac{x + 2y}{x}$</td>
<td>7.</td>
<td>$\frac{3x - 1}{2x - 3}$</td>
<td>10.</td>
<td>$\frac{(2x - 3)}{(x - 4)}$</td>
<td>13.</td>
<td>$\frac{x^2 - 2x + 4}{3x - 4}$</td>
<td></td>
</tr>
<tr>
<td>2.</td>
<td>$\frac{-9c^2}{4b}$</td>
<td>5.</td>
<td>$\frac{-2x}{y}$</td>
<td>8.</td>
<td>$\frac{3x + 5}{x}$</td>
<td>11.</td>
<td>$\frac{(x - 2)}{2x}$</td>
<td>14.</td>
<td>$\frac{9x^2 + 3x + 1}{2(x + 1)}$</td>
<td></td>
</tr>
<tr>
<td>3.</td>
<td>$\frac{x(3x + y)}{y^2(x - 2y)}$</td>
<td>6.</td>
<td>$\frac{x + 2}{x - 7}$</td>
<td>9.</td>
<td>$\frac{-(x + 5)}{(2 + x)}$</td>
<td>12.</td>
<td>$\frac{-(x + 4)}{(3 + 2x)}$</td>
<td></td>
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<td></td>
</tr>
</tbody>
</table>

### Multiplying and Dividing Rational Expressions-Answers Page 95

|   |   |   |   |   |   |   |   |   |   |
|---|---|---|---|---|---|---|---|---|
| 1. | $\frac{4bx^2}{3a^5y^3}$ | 3. | $\frac{2}{5y}$ | 6. | $\frac{-(x + 2)}{5x}$ | 9. | $\frac{x^2 + 2x + 4}{x + 3}$ |
| 2. | $\frac{35b^3x^3}{3a^4y^2}$ | 4. | $-5xy^2$ | 7. | $\frac{-(x + 3)}{2x - 1}$ | 10. | $\frac{x^2 + x + 1}{4x^2}$ |
| 5. | $\frac{2x - 3}{x + 8}$ | 8. | $3x + 2$ |   |   |   |   |
# Adding and Subtracting Rational Expressions—Answers Page 99

1. \( \frac{5}{x} \)  
2. \( -\frac{7n}{xy} \)  
3. \( \frac{-2x-5}{4x-7} \)  
4. \( \frac{3x-8}{2x^2+x-11} \)  
5. \( x+1 \)  
6. \( \frac{10x}{y-2} \)  
7. \( x+4 \)  
8. \( 3(x+1) \)  
9. \( \frac{16y+15x}{10xy} \)  
10. \( \frac{9x+28y}{12x^2y^2} \)  
11. \( \frac{7x+19}{(x+2)(x+3)} \)  
12. \( \frac{-7x-34}{(2x+5)(x-3)} \)  
13. \( \frac{-x^2+17x+3}{(2x-3)(x+6)} \)  
14. \( \frac{8x^2+6x-7}{(2x+5)(2x-3)} \)  
15. \( \frac{7x+11}{(x+1)(x+3)(x-5)} \)

# Division of Polynomials—Answers Page 103

1. \( 6x-5 + \frac{1}{4x-7} \)  
2. \( 8x-4 + \frac{5}{5x-3} \)  
3. \( x^2-8x+7 + \frac{2}{2x+3} \)  
4. \( 3x^2-2 + \frac{5}{4x+7} \)  
5. \( 2x+3 + \frac{2}{2x-3} \)  
6. \( 2x^2+3x+4 + \frac{5}{2x-3} \)  
7. \( 3x^3+x-3 + \frac{1}{2x+6} \)  
8. \( 2x^3-x+4 + \frac{2x-1}{x^2+x-3} \)  
9. \( x-1 + \frac{3}{x-2} \)  
10. \( 2x^3-x^2+7 + \frac{2}{x-1} \)  
11. \( x^2-2x-3 + \frac{2}{x+2} \)  
12. \( 3x^2-x-1 + \frac{4}{x+4} \)  
13. \( x^3-4x^2+3x-5 + \frac{15}{x+2} \)  
14. \( 2x^3-x^2+7 + \frac{2}{x-1} \)  
15. \( 2x^2-x-4 + \frac{6}{x-3} \)
Complex Fractions—Answers Page 107

1. \( \frac{16}{63} \)  
2. \( \frac{6y}{5x^3} \)  
3. \( \frac{3y}{4x^2} \)  
4. 1  
5. \( \frac{2x + 3}{x - 7} \)  
6. \( \frac{xy(x + y)}{2} \)  
7. \( \frac{xy}{x + y} \)  
8. \( \frac{2x + 1}{4x - 3} \)  
9. \( \frac{xy}{y} \)  
10. \( \frac{-(3x + 1)}{9x^2} \)  
11. \( \frac{-(x + 3)}{3x^2} \)  
12. \( \frac{-5}{xy} \)  
13. \( \frac{-2}{2 + h} \)  
14. \( \frac{6x + 7}{-4x + 3} \)

Rational Equations-Answers Page 111

1. \( \left\{ -\frac{4}{3} \right\} \)  
2. \( \{2\} \)  
3. \( \{-7\} \)  
4. \( \{-3\} \)  
5. \( \{4\} \)  
6. \( \left\{ \frac{3}{2} \right\} \)  
7. \( \emptyset \)  
8. \( \left\{ \frac{3}{4} \right\} \)  
9. \( \{-3, 2\} \)  
10. \( \left\{ -\frac{5}{3}, \frac{3}{2} \right\} \)  
11. \( \left\{ \frac{7}{2} \right\} \)  
12. \( \left\{ -\frac{4}{3} \right\} \)  
13. \( \left\{ -\frac{5}{3}, -1 \right\} \)  
14. \( \left\{ -\frac{5}{3}, \frac{4}{3} \right\} \)  
15. \( \emptyset \)  
16. \( \left\{ -\frac{5}{2}, 3 \right\} \)  
17. \( \left\{ -1, \frac{16}{3} \right\} \)  
18. \( \{-1, 18\} \)

Applications-Rational Equations-Answers Page 115

1. \( \frac{3}{4} \) or \( \frac{4}{5} \)  
2. 3 and 4  
3. 6 and 8  
4. \( 1 \frac{1}{5} \) hours  
5. \( 1 \frac{1}{5} \) hours  
6. 20 hours  
7. 12 days
Radicals-Answers Page 121

1. 10
2. \(\frac{11}{2}\)
3. 3
4. \(-4\)
5. not real
6. \(\frac{2}{3}\)
7. 4
8. 13
9. \(\frac{1}{2}\)
10. \(\frac{2}{3}\)
11. not real
12. \(-2\)
13. \(\frac{10}{7}\)
14. \(\frac{3}{4}\)
15. 3
16. \(\sqrt[4]{49}\)
17. \(\sqrt[3]{-5}\)
18. \(\sqrt[4]{64x^3y^3}\)
19. \(\sqrt{2x}\)
20. \(7^{\frac{1}{2}}\)
21. \(x^{\frac{5}{3}}\)
22. \(k\)
23. \((7x^3y)^3 = 343x^9y^3\)
24. \(a^s\)

Rational Exponents-Answers Page 125

1. 6
2. \(\frac{5}{7}\)
3. not real
4. \(-2\)
5. not real
6. \(-2\)
7. 4
8. \(-\frac{2}{5}\)
9. \(-\frac{3}{2}\)
10. \(-\frac{1}{2}\)
11. 32
12. \(-4\)
13. \(-\frac{1}{64}\)
14. \(\frac{1}{7}\)
15. \(\frac{1}{125}\)
16. not real
17. \(-\frac{1}{10}\)
18. \(\frac{8}{27}\)
19. 5
20. 3
21. \(x^{\frac{5}{6}}\)
22. \(16x^{\frac{11}{6}}\)
23. \(\frac{2}{3x^{\frac{11}{12}}}\)
24. \(-\frac{3x^{\frac{8}{3}}}{2y^{\frac{5}{2}}}\)
25. \(10x^{\frac{1}{2}}\)
26. \(\frac{9y}{4x^{\frac{7}{3}}}\)
27. \(\frac{3y^{\frac{1}{6}}}{5x^{\frac{1}{2}}}\)
28. \(-\frac{35}{x^{\frac{5}{12}}}\)
Simplify Radical Expressions-Answers Page 129

1. $4\sqrt{3}$
2. $4\sqrt{5}$
3. $2\sqrt{2}$
4. $2\sqrt{5}$
5. $4x^2\sqrt{2x}$
6. $xy$
7. $4x^2y^2\sqrt{2x}$
8. $12x^3y^3z^4\sqrt{xy}$
9. $3x^3z^3\sqrt{3y^2z}$
10. $\frac{\sqrt{3}}{3}$
11. $\frac{1}{2}$
12. $\frac{\sqrt{15}}{5}$
13. $\frac{\sqrt{6}}{2}$
14. $\frac{5\sqrt{x}}{x}$
15. $\frac{5\sqrt{2x}}{4x}$
16. $\frac{\sqrt{2}}{2}$
17. $\frac{\sqrt{4}}{2}$
18. $\frac{\sqrt{2x}}{3x}$
19. $\frac{\sqrt{21x}}{7x}$
20. $\frac{\sqrt{6xy}}{3y^2}$
21. $\frac{2x^2\sqrt{10}}{5y}$
22. $\frac{\sqrt[3]{7x^2}}{2x}$
23. $\frac{2\sqrt[3]{18x}}{3x}$
24. $\frac{\sqrt[3]{75x^2y^2}}{5x^2}$
25. $\frac{7x\sqrt{xy}}{3y^2}$

Adding and Subtracting Radical Expressions-Answers Page 133

1. $-10\sqrt{7}$
2. 0
3. $-7x\sqrt[3]{10y}$
4. $7\sqrt{3}$
5. $15\sqrt{2}$
6. $5\sqrt{4}$
7. $7\sqrt{3x}$
8. $12x\sqrt{2x}$
9. $7\sqrt{3x} - 13\sqrt{6y}$
10. $2x^2\sqrt{5} - 3x\sqrt{5} + 20\sqrt{5}$
11. $\frac{11\sqrt{2}}{2}$
12. $\frac{23\sqrt{5}}{5}$
13. $\frac{-5\sqrt{4}}{2}$
14. $\frac{14\sqrt{3}}{3}$
15. $\frac{-7\sqrt{2}}{2}$
### Operations with Radical Expressions Page 137

1. $2\sqrt{3} + \sqrt{15}$
2. $3\sqrt{x} + x$
3. $4\sqrt{2} - 6 + 2\sqrt{14} - 3\sqrt{7}$
4. $5\sqrt{2} + \sqrt{70} + \sqrt{10} + \sqrt{14}$
5. $6\sqrt{6} - 12\sqrt{21} + 12\sqrt{10} - 24\sqrt{35}$
6. $8x + 22\sqrt{xy} + 15y$
7. $126 - 36\sqrt{6}$
8. $4x + 28\sqrt{x} + 49$
9. $x + 11 - 6\sqrt{x} + 2$
10. $2$
11. $69$
12. $32x - 87 + 24\sqrt{2x - 6}$
13. $2 - \sqrt{3}$
14. $\sqrt{5} - \sqrt{3}$
15. $\frac{x\sqrt{x} - x\sqrt{y}}{x - y}$
16. $7 + 4\sqrt{3}$
17. $\frac{x + 9\sqrt{x} + 20}{x - 16}$
18. $\frac{3\sqrt{2} + 3 + 2\sqrt{3} + \sqrt{6}}{3}$
19. $\frac{x + 2\sqrt{xy} + y}{x - y}$
20. $\frac{-29 + 17\sqrt{3}}{2}$

### Radical Equations-Answers Page 141

1. $\{4\}$
2. $\{3, -3\}$
3. $\emptyset$
4. $\{21\}$
5. $\left\{\frac{4}{3}, 2\right\}$
6. $\emptyset$
7. $\{8\}$
8. $\{1\}$
9. $\left\{\frac{2}{3}\right\}$
10. $\emptyset$
11. $\{-2\}$
12. $\{8\}$
13. $\left\{\frac{4}{3}\right\}$
14. $\{-1\}$
15. $\{5\}$
16. $\{-7, 3\}$
17. $\left\{\frac{1}{2}, 2\right\}$
18. $\{15\}$
19. $\{11\}$

### Complex Numbers-Answers Page 145

1. $7 - 3i$
2. $-8 - 3i$
3. $21 + 63i$
4. $14 + 8i$
5. $22 - 21i$
6. $-2 - 26i$
7. $29$
8. $4$
9. $-5 + 12i$
10. $2 + 2i\sqrt{3}$
11. $-\frac{5}{3}i$
12. $3 - 5i$
13. $\frac{1}{2} - 2i$ or $\frac{1 - 4i}{2}$
14. $\frac{2 - i}{10}$
15. $\frac{6 + 9i}{13}$
16. $\frac{3 + i}{5}$
17. $\frac{10 + 6i}{17}$
18. $\frac{1 - 3i}{2}$
19. $7 - 5i$
20. $\frac{18 - 13i}{17}$
**Quadratic Formula pg. 151**

1. \( x = \frac{-9 \pm 3\sqrt{3} \cdot 1}{2} \)

2. \( x = \frac{-1 \pm \sqrt{19}}{9} \)

3. \( x = \frac{-5 \pm \sqrt{17}}{8} \)

4. \( x = \frac{-1 \pm i \sqrt{29}}{6} \)

5. \( x = -3 \pm \sqrt{17} \)

6. \( x = \frac{1 \pm 3i \sqrt{7}}{16} \)

7. \( x = \frac{3 \pm 3 \sqrt{5}}{4} \)

8. \( x = \frac{-4 \pm \sqrt{2}}{2} \)

9. \( x = \frac{-1 \pm i \sqrt{2}}{2} \)

10. \( x = \frac{1 \pm \sqrt{11}}{4} \)

11. \( x = \frac{-3 \pm \sqrt{65}}{4} \)

12. \( x = \frac{-3 \pm \sqrt{7}}{2} \)

**Complete Square pg. 155**

1. \( \{ \pm 5 \} \)

2. \( \{ \pm 4 \sqrt{5} \} \)

3. \( \{ \pm 11i \} \)

4. \( \{ \pm 8 \} \)

5. \( \{ \pm \frac{9}{2} \} \)

6. \( \{ \pm \frac{7}{3} \} \)

7. \( \{ \pm \frac{2 \sqrt{10}}{5} \} \)

8. \( \{ \pm \frac{\sqrt{14}}{7} \} \)

9. \( \{ \pm \frac{\sqrt{22}}{2} \} \)

10. \( \{ \pm \frac{\sqrt{6}}{2} \} \)

11. \( \{ \pm \frac{1}{2} \} \)

12. \( \{ \pm 5 \sqrt{3} \} \)

13. \( \{ \pm 3 \} \)

14. \( \{ \pm \frac{5 \sqrt{2}}{2} \} \)

15. \( \{ \pm \frac{5 \sqrt{33}}{2} \} \)

16. \( \{ \pm \frac{7}{2} \} \)

17. \( \{ \pm 6i \} \)

18. \( \{ \pm 3 \} \)

19. \( \{ \pm 8 \} \)

20. \( \{ \pm 3 \} \)

21. \( \{ \pm \frac{5 \sqrt{7}}{4} \} \)

22. \( \{ \pm \frac{1}{2} \} \)

23. \( \{ \pm \frac{2}{2} \} \)

**Applications pg. 159**

1. 8 and 9; -8 and -9

2. 7 and 9; -7 and -9

3. \( 2 \pm \sqrt{3} \)