6.3 Trig Equations with Multiple Angles

When solving trig equations where the angle is something other than x or θ, you MUST adjust the interval.

For example:

1. Solve the equation over the interval $[0, 2\pi)$: $\sin\frac{x}{2} = \frac{1}{2}$

The given interval is $0 \le x < 2\pi$.

So we must adjust it for the new angle: $\frac{0}{2} \le \frac{x}{2} < \frac{2\pi}{2}$ which is $0 \le \frac{x}{2} < \pi$

2. Solve the equation over the interval $[0, 2\pi)$: $\cos 3x = \frac{1}{2}$

The given interval is $0 \le x < 2\pi$.

So we must adjust it for the new angle: $3(0) \le 3(x) < 3(2\pi)$

which is $0 \le 3x < 6\pi$

Solve the equation for exact solutions over the interval $[0, 2\pi)$: p. 284

9) $\cos 2x = -\frac{1}{2}$

$2x = \cos^{-1}\left(-\frac{1}{2}\right)$ π, π

$\frac{2x}{2} = \frac{\frac{2}{3}\pi, \frac{4}{3}\pi, \frac{8}{3}\pi, \frac{10}{3}\pi}{2}$

Add 2π or $\frac{6}{3}\pi$

$x = \frac{1}{3}\pi, \frac{2}{3}\pi, \frac{4}{3}\pi, \frac{5}{3}\pi$

1) Interval

$2(0 \le x < 2\pi)2$

$0 \le 2x < 4\pi$

2) Isolate angle.

3) Find values for angle.

4) Solve for x

10) $\sin 3x = 0$

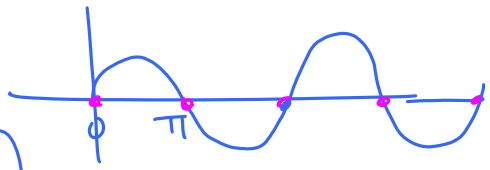
$(0 \le x < 2\pi)3$

$0 \le 3x < 6\pi$

$3x = \sin^{-1}(0)$

$\frac{3x}{3} = \frac{0, \pi, 2\pi, 3\pi, 4\pi, 5\pi}{3}$

$x = 0, \frac{\pi}{3}, \frac{2}{3}\pi, \pi, \frac{4}{3}\pi, \frac{5}{3}\pi$



b) $\cot 3x = \sqrt{3}$ $\quad 0 \le 3x < 6\pi$ $\quad$ Hint $\cot \frac{\pi}{6} = \sqrt{3}$

$3x = \cot^{-1}(\sqrt{3})$ I, III

$\frac{1}{3} \cdot 3x = \left(\frac{\pi}{6}, \frac{7}{6}\pi, \frac{13}{6}\pi, \frac{19}{6}\pi, \frac{25}{6}\pi, \frac{31}{6}\pi\right)\frac{1}{3}$

Add $2\pi = \frac{12}{6}\pi$

$$x = \frac{\pi}{18}, \frac{7}{18}\pi, \frac{13}{18}\pi, \frac{19}{18}\pi, \frac{25}{18}\pi, \frac{31}{18}\pi$$

$3\cos\frac{x}{2} = 2\sqrt{2} - \cos\frac{x}{2}$ $\quad$ (like # 15)

$+1\cos\frac{x}{2} \quad +1\cos\frac{x}{2}$

$\frac{4\cos\frac{x}{2}}{4} = \frac{2\sqrt{2}}{4}$

$\cos\frac{x}{2} = \frac{\sqrt{2}}{2}$

$\frac{x}{2} = \cos^{-1}\left(\frac{\sqrt{2}}{2}\right)$ I, IV

$2 \cdot \frac{x}{2} = \frac{\pi}{4} \cdot 2$

$x = \frac{\pi}{2}$

$\frac{0 \le x < 2\pi}{2}$

$0 \le \frac{x}{2} < \pi$

18. $\cos(2x) - \cos(x) = 0$

Different angles. MUST get the same!

$2\cos^2 x - 1 - \cos x = 0$

Identity:

$\cos 2\theta = 2\cos^2\theta - 1$

$= 1 - 2\sin^2\theta$

$= \cos^2\theta - \sin^2\theta$

$2\cos^2 x - \cos x - 1 = 0$

$(2\cos x + 1)(\cos x - 1) = 0$

$\cos x = -\frac{1}{2} \qquad \cos x = 1$

$x = \cos^{-1}\left(-\frac{1}{2}\right) \qquad x = \cos^{-1}(1)$

$$x = \frac{2}{3}\pi, \frac{4}{3}\pi \qquad x = 0$$

20) $\sin^2\left(\frac{x}{2}\right) - 2 = 0$

$\sqrt{\sin^2\left(\frac{x}{2}\right)} = \sqrt{2}$

$\sin\left(\frac{x}{2}\right) = \pm\sqrt{2} \approx 1.4$

bigger than one!

$\emptyset$

$0 \le x < 2\pi$

$\frac{0 \le x < 2\pi}{2}$

$0 \le \frac{x}{2} < \pi$

Solve the equation for exact solutions over the interval $[0^\circ, 360^\circ)$:

26) $-2\cos 3\theta = \sqrt{3}$

$\cos 3\theta = -\frac{\sqrt{3}}{2}$ II, III

$(0^\circ \le \theta < 360^\circ)3$

$0^\circ \le 3\theta < 1080^\circ$

$3\theta = 150^\circ, 210^\circ, 510^\circ, 570^\circ, 870^\circ, 930^\circ$

Add 360° (+360) ÷3

$\theta = 50^\circ, 70^\circ, 170^\circ, 190^\circ, 290^\circ, 310^\circ$

$\frac{3\cos\theta}{3} = \frac{3\cos 2\theta}{3}$

$\cos\theta = \cos 2\theta$

Diff. angles:

$\cos 2\theta = 2\cos^2\theta - 1$

just like previous problem.

(33-40)

Give all solutions (Hint: adjust the period). Use decimal approximations when necessary.

$3-\cos 2\theta = 4\cos 2\theta$ like # 37

$+\cos 2\theta \quad +\cos 2\theta$

$3 = 5\cos 2\theta$

$\frac{3}{5} = \cos 2\theta$

$\cos^{-1}\left(\frac{3}{5}\right) = 2\theta$

I, IV

I $\frac{53.13^\circ + 360^\circ n}{2} = \frac{2\theta}{2}$ or IV $\frac{306.87^\circ + 360^\circ n}{2} = \frac{2\theta}{2}$

$26.565^\circ + 180^\circ n = \theta$ $\quad$ $153.44^\circ + 180^\circ n = \theta$

$4\csc^2\frac{\theta}{2} = 8\sec\theta$ (like # 35)

$\frac{4}{4} \quad \frac{4}{4}$

$\csc^2\frac{\theta}{2} = 2\sec\theta$

$\frac{1}{\sin^2\left(\frac{\theta}{2}\right)} = \frac{2}{\cos\theta}$

$\cos\theta = 2\sin^2\left(\frac{\theta}{2}\right)$

$\cos\theta = 2\left(\frac{1-\cos\theta}{2}\right)$

$\cos\theta = 1-\cos\theta$

finish

Different Angles:

IDENTITY

$\left(\sin\frac{A}{2}\right)^2 = \left(\pm\sqrt{\frac{1-\cos A}{2}}\right)^2$

$\sin^2\frac{A}{2} = \frac{1-\cos A}{2}$