3.3: Differentiation Formulas

Know these formulas:

1. \( \frac{d}{dx}(c) = 0 \) for any constant \( c \).

2. \( \frac{d}{dx}(x^n) = nx^{n-1} \) for any real number \( n \).

3. \( \frac{d}{dx}[cf(x)] = c \frac{d}{dx}[f(x)] \)

4. \( \frac{d}{dx}[f(x) + g(x)] = \frac{d}{dx}[f(x)] + \frac{d}{dx}[g(x)] \)

5. \( \frac{d}{dx}[f(x) - g(x)] = \frac{d}{dx}[f(x)] - \frac{d}{dx}[g(x)] \)

**Example 1:** Find the derivative of \( f(x) = 7 \).

**Example 2:** Find the derivative of \( f(x) = 5x^3 - x^7 + 12x \).

**Example 3:** Find the derivative of \( g(x) = x^{17} + x^{\frac{1}{2}} \).

Recall:

- \( \sqrt[n]{x} = x^{\frac{1}{n}} \)
- \( \frac{1}{x^n} = x^{-n} \)
Example 4: Find the derivative of \( f(x) = \sqrt[3]{x} + \frac{1}{x^2} \).

Example 5: Find the derivative of \( f(x) = \frac{1}{\sqrt[3]{x}} \).

Example 6: Find the derivative of \( h(x) = \left( \sqrt[4]{x} \right)^5 \).

Example 7: Find the derivative of \( f(x) = -6x^4 \).

Example 8: Find the derivative of \( f(x) = \frac{10}{x^4} \).

Example 9: Find the derivative of \( g(x) = \frac{2\sqrt{x}}{7} \).

Example 10: Find the derivative of \( f(t) = \frac{3}{4t^2} - \sqrt[7]{t} \).
Example 11: Find the derivative of \( f(u) = \frac{7u^5 + u^2 - 9\sqrt{u}}{u^2} \).

Example 12: Find the equation of the tangent line to the graph of \( f(x) = 3x - x^2 \) at the point \((-2, -10)\).

Example 13: Find the point(s) on the graph of \( f(x) = x^2 + 6x \) where the tangent line is horizontal.

7. \[ \frac{d}{dx} \left[ f(x)g(x) \right] = f(x)g'(x) + g(x)f'(x) \] (The Product Rule)

   “the first times the derivative of the second plus the second times the derivative of the first”

8. \[ \frac{d}{dx} \left[ \frac{f(x)}{g(x)} \right] = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2} \] (The Quotient Rule)
Example 14: Find the derivative of $f(x) = x^7(4x^3)$.

Example 15: Find the derivative of $f(x) = (4x^3 + x^2 - 2)(x^4 + 8)$.

Example 16: Find the derivative of $f(x) = \sqrt{x}(x^5 - 3x^2 + 12x)$.

Example 17: Find the derivative of $(4x^3 + 1)(\sqrt{x} + \frac{1}{x} - 2x)$.

Example 18: Find the derivative of $f(x) = \frac{-x^5}{4x^3}$. 
Example 19: Find the derivative of \( g(x) = \frac{4-2x^3}{x^4+3x^2} \).

Example 20: Find the derivative of \( f(x) = \frac{\sqrt{x}}{x^3-x^4} \).

Example 21: Find the derivative of \( f(t) = \frac{7x}{3x+5} + \frac{3x+5}{7x} \).

Definition: The normal line to a curve at the point \( P \) is defined to be the line passing through \( P \) that is perpendicular to the tangent line at that point.

Example 22: Determine the equation of the normal line to the curve \( y = \frac{1}{x} \) at the point \( (3, \frac{1}{3}) \).
Higher order derivatives:

Once the derivative of \( f(x) \) if also a function, it is possible to find the derivative of \( f'(x) \) too. This is called the second derivative and is denoted \( f''(x) \). The second derivative gives the instantaneous rate of change of the derivative. In other words, it tells us how fast the slope is changing.

Similarly, the derivative of \( f''(x) \) can be calculated and this is called the third derivative \( f'''(x) \).

In general, we can keep calculating the derivative of the previous derivative. The \( n \)th derivative is found by taking the derivative \( n \) times. The \( n \)th derivative of \( f \) is denoted \( f^{(n)}(x) \).

Other notation: \( y', y'', y''', \ldots, y^{(n)} \)

\[
\frac{dy}{dx}, \frac{d^2y}{dx^2}, \frac{d^3y}{dx^3}, \ldots, \frac{d^ny}{dx^n}
\]

\( D', D^2, D^3, \ldots, D^n \)

**Example 23:** Suppose that \( y = -7x^5 + 6x^4 - \frac{2}{x} \). Find \( y''' \).

**Example 24:** Suppose that \( f(x) = \sqrt[3]{x} \). Find the second and third derivatives.