1. A fair coin is tossed three times, and we would like to know the probability of getting both a heads and tails to occur. Here are the results of simulating the tosses 24 times: *Fill-in the column at the right with either Yes or No depending on whether both heads and tails occurred or not.*

<table>
<thead>
<tr>
<th>Trial #</th>
<th>First toss</th>
<th>Second toss</th>
<th>Third toss</th>
<th>Did both occur?</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>H</td>
<td>H</td>
<td>T</td>
<td>Yes</td>
</tr>
<tr>
<td>2</td>
<td>H</td>
<td>H</td>
<td>H</td>
<td>No</td>
</tr>
<tr>
<td>3</td>
<td>T</td>
<td>T</td>
<td>H</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>T</td>
<td>T</td>
<td>T</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>H</td>
<td>H</td>
<td>H</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>T</td>
<td>T</td>
<td>T</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>T</td>
<td>T</td>
<td>T</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>T</td>
<td>T</td>
<td>H</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>T</td>
<td>H</td>
<td>T</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>T</td>
<td>H</td>
<td>T</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>H</td>
<td>H</td>
<td>T</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>T</td>
<td>H</td>
<td>T</td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>H</td>
<td>T</td>
<td>H</td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>T</td>
<td>H</td>
<td>H</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>H</td>
<td>T</td>
<td>H</td>
<td></td>
</tr>
<tr>
<td>16</td>
<td>T</td>
<td>H</td>
<td>T</td>
<td></td>
</tr>
<tr>
<td>17</td>
<td>T</td>
<td>H</td>
<td>H</td>
<td></td>
</tr>
<tr>
<td>18</td>
<td>H</td>
<td>H</td>
<td>T</td>
<td></td>
</tr>
<tr>
<td>19</td>
<td>H</td>
<td>H</td>
<td>H</td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>H</td>
<td>T</td>
<td>T</td>
<td></td>
</tr>
<tr>
<td>21</td>
<td>H</td>
<td>H</td>
<td>H</td>
<td></td>
</tr>
<tr>
<td>22</td>
<td>H</td>
<td>T</td>
<td>H</td>
<td></td>
</tr>
<tr>
<td>23</td>
<td>T</td>
<td>T</td>
<td>H</td>
<td></td>
</tr>
<tr>
<td>24</td>
<td>H</td>
<td>T</td>
<td>H</td>
<td></td>
</tr>
</tbody>
</table>

**a)** Use the results to estimate the probability of seeing both heads and tails in three tosses of a fair coin. *(Empirical Probability)*

**b)** Find the exact probability of seeing both heads and tails in three tosses of a fair coin. *(Theoretical Probability)*
2. Spin the spinner.

<table>
<thead>
<tr>
<th>Probability</th>
<th>Chance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probability of red = $\frac{3}{8}$</td>
<td>There are 3 chances in 8 of stopping on red.</td>
</tr>
<tr>
<td>Probability of green = $\frac{2}{8}$ or $\frac{1}{4}$</td>
<td>There are 2 chances in 8 of stopping on green.</td>
</tr>
<tr>
<td>Probability of yellow = $\frac{0}{8}$ or 0</td>
<td>There are 0 chances in 8 of stopping on yellow.</td>
</tr>
</tbody>
</table>

Find each probability.

One of these names is to be drawn from a hat.

<table>
<thead>
<tr>
<th>Name</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mary</td>
</tr>
<tr>
<td>Jenny</td>
</tr>
<tr>
<td>Bob</td>
</tr>
<tr>
<td>Marilyn</td>
</tr>
<tr>
<td>Bill</td>
</tr>
<tr>
<td>Jack</td>
</tr>
<tr>
<td>Jerry</td>
</tr>
<tr>
<td>Tina</td>
</tr>
<tr>
<td>Connie</td>
</tr>
<tr>
<td>Joe</td>
</tr>
</tbody>
</table>

Number of 3-letter names: 2
Total number of names: 10

a) $P(3$-letter name) = $\frac{2}{10}$ or $\frac{1}{5}$

What is the probability of drawing a 3-letter name?

b) $P(4$-letter name) = $\frac{4}{10}$ or $\frac{2}{5}$

c) $P($name starting with B) = $\frac{2}{10}$ or $\frac{1}{5}$

d) $P($name starting with T) =

e) $P($7-letter name) =

f) $P($name starting with S) =

g) $P($name ending with Y) =

One of these cards will be drawn without looking.

<table>
<thead>
<tr>
<th>Card</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
</tr>
<tr>
<td>4</td>
</tr>
<tr>
<td>7</td>
</tr>
<tr>
<td>J</td>
</tr>
<tr>
<td>S</td>
</tr>
<tr>
<td>9</td>
</tr>
<tr>
<td>10</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>M</td>
</tr>
<tr>
<td>5</td>
</tr>
<tr>
<td>4</td>
</tr>
<tr>
<td>J</td>
</tr>
</tbody>
</table>

h) $P(2$ =

number of twos

total number of cards

i) $P(5$ =

j) $P(J$ =

k) $P($a number$) =

l) $P(4$ =

m) $P(T$ =

n) $P($a letter$) =$
3. To find a theoretical probability, first list all possible outcomes. Then use the formula:

\[ P(\text{event}) = \frac{\text{number of favorable outcomes}}{\text{total number of outcomes}}. \]

A letter is selected at random from the letters of the word FLORIDA. What is the probability that the letter is an A?

- There are 7 letters (possible outcomes).
- There is 1 A, which represents a favorable outcome.

\[ P(A) = \frac{\text{number of favorable outcomes}}{\text{total number of outcomes}} = \frac{1}{7} \]

The probability that the letter is an A is \( \frac{1}{7} \).

Selecting a letter other than A is called not A and is the complement of the event A. The probabilities of an event and its complement add to 1, or 100%.

What is the probability of the event not A?

\[ P(A) + P(\text{not } A) = 1 \]
\[ \frac{1}{7} + P(\text{not } A) = 1 \]
\[ P(\text{not } A) = 1 - \frac{1}{7} = \frac{6}{7} \]

The probability of the event not A (selecting F, L, O, R, I, or D) is \( \frac{6}{7} \).

Use the spinner. Write each probability as a fraction. Then write it as a decimal and a percent.

a) \( P(5) \)

\[ = \frac{1}{5} \]

b) \( P(\text{odd number}) \)

\[ = \frac{2}{5} \]

A box contains cards numbered from 1 to 10. Write each probability as a fraction, a decimal, and a percent.

c) \( P(\text{even number}) \)

d) \( P(\text{number less than 4}) \)

e) \( P(\text{not 5}) \)


f) \( P(M) \)

g) \( P(\text{not vowel}) \)

h) \( P(\text{not E}) \)

A number is selected at random from the numbers 1 to 50. Find each probability.

i) \( P(\text{multiple of 3}) \)

j) \( P(\text{a factor of 50}) \)

k) \( P(\text{not factor of 50}) \)
4. A spinner numbered 1 through 10 is spun. Each outcome is equally likely. Write each probability as a fraction, decimal, and percent.

a) \( P(9) \)  

b) \( P(\text{even}) \)  

c) \( P(\text{number greater than 0}) \)  

d) \( P(\text{multiple of 4}) \)  

There are eight blue marbles, nine orange marbles, and six yellow marbles in a bag. It is equally likely that any marble is drawn from the bag.

e) Find the probability of drawing a blue marble.  

f) Find the probability of drawing a yellow marble.  

g) What marble could you add or remove so that the probability of drawing a blue marble is \( \frac{1}{3} \)?

Suppose you have a box that contains 12 slips of paper as shown. Each slip of paper is equally likely to be drawn. Find the probability of each event.

h) \( P(\text{red}) \)  

i) \( P(\text{blue}) \)  

ej) \( P(\text{yellow}) \)  

k) \( P(\text{red}) + P(\text{blue}) \)  

l) \( P(\text{red}) + P(\text{yellow}) \)  

m) \( P(\text{blue}) + P(\text{yellow}) \)  

n) \( P(\text{red or blue}) \)  

o) \( P(\text{red or yellow}) \)  

p) \( P(\text{blue or yellow}) \)  

q) \( P(\text{not red}) \)  

r) \( P(\text{not blue}) \)  

s) \( P(\text{not yellow}) \)
5. Determine the following probability values. Convert them into decimals rounded to the nearest hundredth.

Spin the spinner once. Find the following probabilities.

a) \( P(9) \)   

b) \( P(\text{multiple of 2}) \)   

c) \( P(\text{even number}) \)   

d) \( P(\text{prime number}) \)   

e) \( P(\text{number < 8}) \)   

f) \( P(\text{factor of 8}) \)   

A laundry basket contains 3 red socks, 5 orange socks, 4 blue socks, and 8 black socks. Without looking, choose a sock. Find the following probabilities.

g) \( P(\text{orange}) \)   

h) \( P(\text{blue}) \)   

i) \( P(\text{not blue}) \)   

j) \( P(\text{white}) \)   

You roll a fair die and toss a fair coin simultaneously. Find the following probabilities.

k) \( P(1, \text{heads}) \)   

l) \( P(2, \text{tails}) \)   

m) \( P(6, \text{heads or tails}) \)   

n) \( P(\text{even number, tails}) \)   

o) \( P(\text{odd number, heads or tails}) \)
6. Two fair dice are rolled. Find the following conditional probabilities of rolling:

<table>
<thead>
<tr>
<th>First die</th>
<th>Second die</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1,1)</td>
<td>(1,2)</td>
</tr>
<tr>
<td>(1,3)</td>
<td>(1,4)</td>
</tr>
<tr>
<td>(1,5)</td>
<td>(1,6)</td>
</tr>
<tr>
<td>(2,1)</td>
<td>(2,2)</td>
</tr>
<tr>
<td>(2,3)</td>
<td>(2,4)</td>
</tr>
<tr>
<td>(2,5)</td>
<td>(2,6)</td>
</tr>
<tr>
<td>(3,1)</td>
<td>(3,2)</td>
</tr>
<tr>
<td>(3,3)</td>
<td>(3,4)</td>
</tr>
<tr>
<td>(3,5)</td>
<td>(3,6)</td>
</tr>
<tr>
<td>(4,1)</td>
<td>(4,2)</td>
</tr>
<tr>
<td>(4,3)</td>
<td>(4,4)</td>
</tr>
<tr>
<td>(4,5)</td>
<td>(4,6)</td>
</tr>
<tr>
<td>(5,1)</td>
<td>(5,2)</td>
</tr>
<tr>
<td>(5,3)</td>
<td>(5,4)</td>
</tr>
<tr>
<td>(5,5)</td>
<td>(5,6)</td>
</tr>
<tr>
<td>(6,1)</td>
<td>(6,2)</td>
</tr>
<tr>
<td>(6,3)</td>
<td>(6,4)</td>
</tr>
<tr>
<td>(6,5)</td>
<td>(6,6)</td>
</tr>
</tbody>
</table>

a) a sum of 8, given that the sum is greater than 7.

b) a sum of 6, given that each die shows the same number.

c) each die shows the same number, given that the sum is 9.

7. Two cards are drawn at random without replacement from an ordinary deck, find the following conditional probabilities:

a) the second card is a heart, given that the first card is a heart

b) the second card is black, given that the first card is a club
8. Michele Jordache has a large collection of basketball shoes. The table below shows how many of each type she has.

<table>
<thead>
<tr>
<th></th>
<th>Red</th>
<th>White</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hi Top</td>
<td>16</td>
<td>22</td>
<td>38</td>
</tr>
<tr>
<td>Lo Cut</td>
<td>9</td>
<td>3</td>
<td>12</td>
</tr>
<tr>
<td>Total</td>
<td>25</td>
<td>25</td>
<td>50</td>
</tr>
</tbody>
</table>

a) How many pairs does she have? _____

b) How many pairs are red? _____

c) How many pairs are Lo Cut? _____

Before each game, she picks a pair at random.

d) \( P(\text{white}) = \)

e) \( P(\text{lo cut}) = \)

f) \( P(\text{white and lo cut}) = \)

g) \( P(\text{white or lo cut}) = \)

h) \( P(\text{white} \mid \text{lo cut}) = \)

i) \( P(\text{hi top} \mid \text{red}) = \)

j) \( P(\text{red} \mid \text{hi top}) = \)

k) \( P(\text{white} \mid \text{red}) = \)
9. Suppose that \( P(E) = .3 \), \( P(F) = .5 \), and \( P(E \cap F) = .15 \).
a) Complete the following probability diagram:

\[
\begin{align*}
&\text{E} \\
&\text{F} \\
&\text{S}
\end{align*}
\]

b) \( P(E \cup F) \)  

c) \( P(E | F) \)  

d) \( P(F | E) \)  

e) Are \( E \) and \( F \) independent?

10. A shop that produces cabinets has two employees: Steve and Casey. 95% of Casey’s work is satisfactory, and 10% of Steve’s work is unsatisfactory. 60% of the shop’s work is made by Casey.
a) Complete the following probability tree.

\[
\begin{align*}
\text{Satisfactory} \\
&\text{Steve} \\
&\quad \text{Unsatisfactory} \\
&\quad \quad .1 \\
&\quad \text{Satisfactory} \\
&\quad \quad .95 \\
&\text{Casey} \\
&\quad \text{Unsatisfactory}
\end{align*}
\]

b) \( P(\text{Satisfactory}) \)  

c) \( P(\text{Unsatisfactory}) \)  

d) \( P(\text{Casey and Satisfactory}) \)

e) \( P(\text{Casey|Satisfactory}) \)  

f) \( P(\text{Steve and Un satisfactory}) \)  

g) \( P(\text{Steve|Un satisfactory}) \)
11. The following table gives the results of 1,000 weather forecasts. If one forecast is chosen at random, determine the following.

<table>
<thead>
<tr>
<th></th>
<th>Rain</th>
<th>No Rain</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Forecast of Rain</td>
<td>66</td>
<td>156</td>
<td>222</td>
</tr>
<tr>
<td>Forecast of No Rain</td>
<td>14</td>
<td>764</td>
<td>778</td>
</tr>
<tr>
<td>Total</td>
<td>80</td>
<td>920</td>
<td>1000</td>
</tr>
</tbody>
</table>

a) \( P(\text{Rain}) \)  

b) \( P(\text{Forecast of Rain}) \)  

c) \( P(\text{Rain and Forecast of Rain}) \)

d) \( P(\text{Rain|Forecast of Rain}) \)  

e) \( P(\text{No Rain|Forecast of No Rain}) \)

f) Are Forecast of Rain and No Rain independent?

12. Five students, Art, Bonnie, Carol, Doug, and Ed, volunteer to sell refreshments at the faculty-student basketball game, and only three students are needed. In order to select three students, the following procedure is to be used. Each of the ten possible selections of three students (listed below) is written on a piece of paper, and then one piece of paper is selected at random.

Art, Bonnie, Carol  
Art, Bonnie, Doug  
Bonnie, Carol, Doug  
Art, Bonnie, Ed  
Art, Carol, Doug  
Bonnie, Carol, Ed  
Art, Carol, Ed  
Art, Doug, Ed  
Bonnie, Doug, Ed  
Carol, Doug, Ed

a) What is the probability that Art is selected?

b) What is the probability that Doug is not selected?

c) What is the probability that both Art and Ed are selected?

d) If Bonnie is selected, what is the probability that Carol is not selected?

e) What is the probability that either Bonnie or Carol is selected?
Ten students, Art, Bonnie, Carol, Doug, Ed, Frank, George, Hank, Ivan, and Jeff, volunteer to sell refreshments at the faculty-student basketball game, and only three students are needed. In order to select three students, the following procedure is to be used. Each of the 120 possible selections of three students is written on a piece of paper, and then one piece of paper is selected at random.

a) What is the probability that Art is selected?

b) What is the probability that Doug is not selected?

c) What is the probability that both Art and Ed are selected?

d) If Bonnie is selected, what is the probability that Carol is not selected?

e) What is the probability that either Bonnie or Carol is selected?
14. Use a calculator, paper and pencil, or mental math to evaluate each factorial.

<table>
<thead>
<tr>
<th>a) 6!</th>
<th>b) 12!</th>
<th>c) 9!</th>
<th>d) $\frac{8!}{5!}$</th>
<th>e) $\frac{12!}{3!}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>f) $gP_5$</td>
<td>g) $gP_2$</td>
<td>h) $10P_8$</td>
<td>i) $5P_5$</td>
<td>j) $15P_6$</td>
</tr>
</tbody>
</table>

Solve.

k) In how many ways can all the letters of the word WORK be arranged?

m) A disk jockey can play eight songs in one time slot. In how many different orders can the eight songs be played?

o) At a track meet, 42 students entered the 100-m race. In how many ways can first, second, and third places be awarded?

q) A car dealer has 38 used cars to sell. Each day two cars are chosen for advertising specials. One car appears in a television commercial and the other appears in a newspaper advertisement. In how many ways can the two cars be chosen?

s) A certain type of luggage has room for three initials. How many different 3-letter arrangements of letters are possible?

t) A roller coaster has room for 10 people. The people sit single file, one after the other. How many different arrangements are possible for 10 passengers on the roller coaster?

l) In how many ways can you arrange seven friends in a row for a photo?

n) Melody has nine bowling trophies to arrange in a horizontal line on a shelf. How many arrangements are possible?

p) In how many ways can a president, a vice president, and a treasurer be chosen from a group of 15 people running for office?

r) A bicycle rack outside a classroom has room for six bicycles. In the class, 10 students sometimes ride their bicycles to school. How many different arrangements of bicycles are possible for any given day?
15.
Compute each number of combinations.

a) \(_{9}C_{1}\)  
b) \(_{8}C_{4}\)  
c) \(_{11}C_{4}\)  
d) \(_{11}C_{7}\)

----------  

----------  

----------  

----------

e) \(_{4}C_{4}\)  
f) \(_{9}C_{3}\)  
g) \(_{12}C_{6}\)  
h) \(_{8}C_{2}\)

----------  

----------  

----------  

----------
i) 3 videos from 10

----------  
j) 2 letters from

LOVE

----------  
k) 4 books from 8

----------  
l) 5 people from 7

----------

Solve.

m) Ten students from a class have volunteered to be on a committee to organize a dance. In how many ways can six be chosen for the committee?

----------

----------

----------

----------

o) A team of nine players is to be chosen from 15 available players. In how many ways can this be done?

----------

----------

----------

----------

q) At a party there are 12 people present. The host requests that each person present shake hands exactly once with every other person. How many handshakes are necessary?

----------

----------

----------

----------

r) In a talent show, five semi-finalists are chosen from 46 entries. In how many ways can the semi-finalists be chosen?

----------

----------

----------

----------

s) Five friends, Billi, Joe, Eduardo, Mari, and Xavier, want one photograph taken of each possible pair of friends. Use B, J, E, M, and X, and list all of the pairs that need to be photographed.

----------

----------

----------

----------

t) Choose A, B, C, or D. Which situation described has \(_{8}C_{3}\) possible outcomes?

A. Select three letters from 8 to form a 3-letter password.

B. Find the possible ways that first, second, and third prize winners can be selected from 8 contestants.

C. Arrange 8 people in 3 rows.

D. Pick a team of 3 people from 8 players.
16. 
Simplify each expression.

a) \(7P_2\) 
\[\text{__________}\]

b) \(7C_2\) 
\[\text{__________}\]

c) \(8P_3\) 
\[\text{__________}\]

d) \(9P_4\) 
\[\text{__________}\]

e) \(3C_2\) 
\[\text{__________}\]

f) \(10C_4\) 
\[\text{__________}\]

g) Art, Becky, Carl, and Denise are lined up to buy tickets.
   a. How many different permutations of the four are possible?
      \[\text{___________________________}\]

   b. Suppose Ed was also in line. How many permutations would there be?
      \[\text{___________________________}\]

c. In how many of the permutations of the five is Becky first?
   \[\text{___________________________}\]

d. What is the probability that a permutation of this five chosen at random will have Becky first?
   \[\text{___________________________}\]

h) Art, Becky, Carl, Denise, and Ed all want to go to the concert. However, there are only 3 tickets. How many ways can they choose the 3 who get to go to the concert?
   \[\text{___________________________}\]

i) A combination lock has 36 numbers on it. How many different 3-number combinations are possible if no number may be repeated?
   \[\text{___________________________}\]

Numbers are to be formed using the digits 1, 2, 3, 4, 5, and 6. No digit may be repeated.

j) How many two-digit numbers can be formed?  
   \[\text{__________}\]

k) How many three-digit numbers can be formed?  
   \[\text{__________}\]

l) How many four-digit numbers can be formed?  
   \[\text{__________}\]

m) How many five-digit numbers can be formed?  
   \[\text{__________}\]

n) How many six-digit numbers can be formed?  
   \[\text{__________}\]
17. Choose a calculator, paper and pencil, or mental math.

a) How many license plates are possible if four letters are to be followed by two digits?

________________________

b) How many license plates are possible if two letters are to be followed by four digits?

________________________

c) A dress pattern offers two styles of skirts, three styles of sleeves, and four different collars. How many different types of dresses are available from one pattern?

________________________

d) In a class of 250 eighth graders, 14 are running for president, 12 are running for vice president, 9 are running for secretary, and 13 are running for treasurer. How many different results are possible for the class election?

________________________

e) A home alarm system has a 3-digit code that can be used to deactivate the system. If the homeowner forgets the code, how many different codes might the homeowner have to try?

________________________

f) A 4-letter password is required to enter a computer file. How many passwords are possible if no letter is repeated and nonsense words are allowed?

________________________

18. *Numbers* is a game in which you bet $1 on any three-digit number from 000 to 999. If your number is randomly selected, you get $500.

<table>
<thead>
<tr>
<th>winnings</th>
<th>–$1</th>
<th>$499</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P(\text{winnings}) )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Expected Value =**
19. In Keno, the house has a pot containing 80 balls, each marked with a different number from 1 to 80. You buy a ticket for $1 and mark one of the 80 numbers on it. The house then selects 20 numbers at random. If your number is among the 20, you get $3.20.

<table>
<thead>
<tr>
<th>winnings</th>
<th>−$1</th>
<th>$2.20</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P(winnings)$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

You win the $3.20 if your number is one of the 20 numbers selected. The probability that your number is the first number selected is \( \frac{1}{80} \); the probability that your number is the second number selected is \( \frac{1}{80} \); and so forth. So the probability that your number is one of the 20 numbers selected is \( \frac{1}{80} + \frac{1}{80} + \cdots + \frac{1}{80} \) \(20 \text{ times}\).

Expected Value =
Complete the **probability distribution** and **histogram** for the following random variables, and determine the **expected value**.

20. Two fair dice are rolled and the random variable, $x$, is the sum of the faces showing.

<table>
<thead>
<tr>
<th>First die</th>
<th>Second die</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1,1)</td>
<td>(1,2)</td>
</tr>
<tr>
<td>(1,3)</td>
<td>(1,4)</td>
</tr>
<tr>
<td>(1,5)</td>
<td>(1,6)</td>
</tr>
<tr>
<td>(2,1)</td>
<td>(2,2)</td>
</tr>
<tr>
<td>(2,3)</td>
<td>(2,4)</td>
</tr>
<tr>
<td>(2,5)</td>
<td>(2,6)</td>
</tr>
<tr>
<td>(3,1)</td>
<td>(3,2)</td>
</tr>
<tr>
<td>(3,3)</td>
<td>(3,4)</td>
</tr>
<tr>
<td>(3,5)</td>
<td>(3,6)</td>
</tr>
<tr>
<td>(4,1)</td>
<td>(4,2)</td>
</tr>
<tr>
<td>(4,3)</td>
<td>(4,4)</td>
</tr>
<tr>
<td>(4,5)</td>
<td>(4,6)</td>
</tr>
<tr>
<td>(5,1)</td>
<td>(5,2)</td>
</tr>
<tr>
<td>(5,3)</td>
<td>(5,4)</td>
</tr>
<tr>
<td>(5,5)</td>
<td>(5,6)</td>
</tr>
<tr>
<td>(6,1)</td>
<td>(6,2)</td>
</tr>
<tr>
<td>(6,3)</td>
<td>(6,4)</td>
</tr>
<tr>
<td>(6,5)</td>
<td>(6,6)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$x$</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P(x)$</td>
<td>$\frac{1}{36}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$\frac{1}{36}$</td>
</tr>
</tbody>
</table>

$E(x) =$
21. Two names are randomly drawn from a hat without replacement. Three of the names in the hat are Aggies, and the other two are Longhorns. Let the random variable, $x$, be the total number of Aggies selected.

The sample space for this experiment is

$$\{L_1, A_1\}, \{L_1, A_2\}, \{L_1, A_3\}, \{L_2, A_1\}, \{L_2, A_2\}, \{L_2, A_3\}, \{A_1, A_2\}, \{A_1, A_3\}, \{A_2, A_3\}, \{L_1, L_2\}$$

<table>
<thead>
<tr>
<th>$x$</th>
<th>0</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P(x)$</td>
<td>$\frac{1}{10}$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$E(x) =$
22. The probability density function for a continuous random variable $X$, which takes on values from 1 to 5 inclusive, is given by the following graph.

![Graph of the probability density function]

a) Verify that the probability density function is valid by showing that the total area under its graph is 1.

b) Find $P(X > 4.5)$

c) Find $P(1.8 < X < 4.2)$

d) Using the idea of a balance point, what’s the mean or expected value of $X$?

e) Using the idea that the median separates the upper 50% of the values from the lower 50% of the values, what’s the median of $X$?
23. The probability density function for a continuous random variable $X$, which takes on values from 1 to 5 inclusive, is given by the following graph.

a) Verify that the probability density function is valid by showing that the total area under its graph is 1.

b) Find $P(X > 4)$

c) Find $P(2 < X < 4)$

d) Using the idea of a balance point, what’s the mean or expected value of $X$?

e) Using the idea that the median separates the upper 50% of the values from the lower 50% of the values, what’s the median of $X$?

f) Using the idea that the mode is the most likely value to occur, what’s the mode of $X$?
24. The probability density function for a continuous random variable $X$, which takes on values from 1 to 5 inclusive, is given by the following graph.

![Graph of the probability density function]

a) Verify that the probability density function is valid by showing that the total area under its graph is 1.

b) Find $P(X < 2)$

c) Find $P(2 < X < 4)$

d) Using the idea that the median separates the upper 50% of the values from the lower 50% of the values, what’s the median of $X$?

e) Using the idea that the mode is the most likely value to occur, what’s the mode of $X$?
25. A fair die is rolled until all six faces occur, and we would like to know the expected number of rolls required for this to happen. Here are the results of simulating the rolls 12 times:

<table>
<thead>
<tr>
<th>Trial #</th>
<th># of rolls to get all 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>8</td>
</tr>
</tbody>
</table>

Complete the table, and use the results to estimate the expected number of rolls required to see all six numbers with a fair die.
Do the socks tell the story?

A sock drawer can tell a lot about probability. Just ask a question about what might happen when someone reaches in for a pair of socks and write a ratio about probable outcomes. Solve each probability problem with a ratio. Look in the sock drawer (below). Find all the socks with that solution. Color the sock in the color shown for the section.

Color the answer-socks RED for problems 1 and 2.
A dryer holds 12 socks. Eight are black (B). One is red (R). The rest are white (W). Jerome reaches in (without looking) and grabs one.
1. \( P (W) = \)
2. \( P (\text{not B}) = \)

Color the answer-socks BLUE for problems 3 and 4.
Trevor’s soccer bag holds lots of socks: 4 red (R), 7 green (G), 3 white (W), and 2 purple (P).
3. He takes 2 socks. \( P (\text{R and G}) = \)
4. He replaces 2 and takes 3. \( P (\text{3W}) = \)

Color the answer-socks GREEN for problems 5–7.
A laundry basket holds 21 socks. Nine are green (G). Two are red (R). The rest are black (B). Michelle grabs two socks.
5. \( P (\text{pair of B}) = \)
6. \( P (\text{G, R}) = \)
7. \( P (\text{pair of G}) = \)

Color the answer-socks YELLOW for problem 8.
8. Abby’s drawer has 18 blue (B) socks and four white (W) socks. How many socks will she have to take out to be certain that she will have a matching pair?
Heard It Through the Grapevine

Directions: Solve each permutation, combination, or factorial. Find each answer in the Letter Box, and notice the letter next to it. Write that letter on the blank space that contains the number of the problem. Some letters have been done for you. The resulting message will be the answer to the riddle.

1. P(6, 3)  8. C(8, 4)
2. P(8, 4)  9. C(5, 5)
3. 7!  10. C(7, 5)
4. P(6, 6)  11. 9!
5. P(7, 5)  12. P(9, 5)
6. C(6, 3)  13. C(9, 5)
7. 6!  14. P(3, 2)

Letter Box

| N = 126 | F = 2520 |
| A = 720 | P = 20  |
| S = 6  | O = 5040 |
| U = 15,120 | G = 70 |
| E = 1680 | H = 120 |
| R = 1  | B = 362,880 |
| K = 21 |

Question: What might the mother and father grapes have to say about their children?

<table>
<thead>
<tr>
<th>W</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
</tr>
<tr>
<td>4</td>
</tr>
<tr>
<td>9</td>
</tr>
<tr>
<td>2</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>14</td>
</tr>
</tbody>
</table>
28.

There is one of ten team cards inside a box of cereal. The teams are equally distributed among the boxes. Estimate how many boxes of cereal you need to purchase to collect all ten teams.

Average number to get all 10:

The exact expected number is about 29.29.

A gas station gives away one of eight drinking glasses each time you buy a tank of gas. There is an equal chance of getting any one of the glasses. Estimate how many tanks you will have to buy to get all eight glasses.

Average number to get all 8:

The exact expected number is about 21.74.

See the Collecting Link on the course Webpage!!!!
A bag contains 3 black and 2 white marbles. A marble is drawn at random and then replaced. Find each probability.

1. $P(2 \text{ blacks})$  
2. $P(\text{black, white})$  
3. $P(\text{white, black})$  
4. $P(2 \text{ whites})$

Each letter from the word MISSISSIPPI is written on a separate slip of paper. The 11 slips of paper are placed in a sack and two slips are drawn at random. The first pick is not replaced.

5. Find the probability that the first letter is M and the second letter is I.  
6. Find the probability that the first letter is I and the second letter is P.  
7. Find the probability that the first letter is S and the second letter is also S.

Solve.

8. On a TV game show, you can win a car by drawing two aces from a standard deck of cards. The first card is not replaced. What is your probability of winning?  
9. You roll a number cube eight times, and each time you roll a 4. What is the probability that on the ninth roll, you will roll a 6?

10. Two letters of the alphabet are chosen randomly without replacement. Find each probability.
   a. $P(\text{both vowels})$  
   b. $P(\text{both consonants})$

11. There are 4 brown shoes and 10 black shoes on the floor. Your puppy carries away two shoes and puts one shoe in the trash can and one shoe in the laundry basket.
   a. Complete the tree diagram to show the probability of each outcome.
   b. What is the probability that there will be a brown shoe in both the trash and the laundry basket?  

12. Use the data at the right to find $P(\text{right-handed male})$ and $P(\text{left-handed female})$ if one person is chosen at random.

<table>
<thead>
<tr>
<th>Male</th>
<th>Female</th>
</tr>
</thead>
<tbody>
<tr>
<td>Right-handed</td>
<td>86</td>
</tr>
<tr>
<td>Left-handed</td>
<td>14</td>
</tr>
<tr>
<td>Total</td>
<td>100</td>
</tr>
</tbody>
</table>