3.4: Present Value of an Annuity; Amortization

Present Value of an Ordinary Annuity

\[ PV = PMT \left[ \frac{1 - (1+i)^{-n}}{i} \right] \]

where

- \( PMT \): periodic payment (made at end of period)
- \( i = \frac{r}{m} \): rate per period
- \( n \): number of payments (periods)
- \( PV \): present value of all payments

**Example 1:** How much should you deposit into an account that pays 6% compounded semiannually so that $1,000 may be withdrawn every 6 months for three years?

The *amortization* of a debt is the process of paying it off in equal installments. For example, if I buy a new car and don’t have the cash for it, I *amortize* the debt by making equal monthly payments.
Example 2: Scott buys a fancy television for $800. He plans to make equal monthly payments for 18 months at 18% compounded monthly.

a. What are his monthly payments?
b. How much total interest will he pay?

Example 3: I buy a car for $15,000. I put $700 down and the dealer gives me $800 for my trade-in. I finance the rest at 5.5% for five years (compounded monthly). How large are my monthly payments? How much total money do I pay for the car? How much interest?
Example 4:  Scott and Jennifer are considering buying a house. The house they like costs $110,000, and they have saved $10,000 for a down payment.

a. What will be their monthly payment for a 30-year loan at 5% (compounded monthly)? How much interest will they pay?

b. What will be their monthly payment for a 15-year loan at 5% (compounded monthly)? How much interest will they pay?

c. What will be their monthly payment for a 15-year loan at 4.6% (compounded monthly)? How much interest will they pay?
Summary of formulas:

**Single Payment**
- **Simple Interest**
  
  \[ A = P(1 + rt) \]

- **Compound Interest**
  
  \[ A = P \left(1 + \frac{r}{m}\right)^n \]

**Sequence of Payments**
- **Involves Value after Payments**
  
  \[ FV = PMT \left[ \frac{(1+i)^n - 1}{i} \right] \]

- **Involves Value before Payments**
  
  \[ PV = PMT \left[ \frac{1 - (1+i)^{-n}}{i} \right] \]