3.2: Compound Interest

If at the end of a payment period, the interest due is reinvested at the same rate, then the interest as well as the principal will earn interest. This is called *compound interest*. The interest is paid into the account at the end of each compounding period.

**Example 1:** Suppose you invest $1000 compounded quarterly at an annual interest rate of 8%. How much money will you have after one year?

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**Compound Interest:**

\[
A = P(1 + i)^n
\]

\[
= P \left(1 + \frac{r}{m}\right)^n
\]

where

\[
i = \frac{r}{m}\] is the interest rate per compounding period
\[r = \text{annual interest rate}\]
\[m = \text{number of compounding periods per year}\]
\[n = \text{total number of compounding periods}\]
\[P = \text{principal (present value)}\]
\[A = \text{amount (future value) at the end of } n \text{ compounding periods.}\]
Example 2: What is the future value of $1000 after 8 years at 6% compounded monthly?

Example 3: How much should I invest now at 4% interest compounded monthly in order to have $10,000 in 6 years?

Example 4: You decide to invest some money so that you will have $1,000,000 on your 75th birthday. At 8% compounded quarterly, how much should you invest on your 25th birthday?
Example 5: How long will it take $5,000 to grow to $7,000 if it is invested at 8% compounded monthly?

Example 6: How long will it take money to double if it is invested at 7.5% compounded monthly?

Effective rates:

The effective rate, sometimes called the annual percentage yield, converts a compound interest rate to an equivalent simple interest rate. This allows us to compare interest rates which have different compounding periods.

Effective Rate:

The effective rate, or annual percentage yield (APY), is given by

\[
 r_e = APY = \left(1 + \frac{r}{m}\right)^m - 1,
\]

where

- \( r = \) annual interest rate
- \( m = \) number of compounding periods per year.
Example 7: What is the effective rate for money invested at 6% compounded quarterly?

Example 8: Which investment is better, Note A at 9% compounded monthly or Note B at 9.2% compounded semiannually?