8.4: Bayes’ Formula

Previously, when we discussed conditional probabilities, we considered the probability of a “later” event (in a probability tree), given that an “earlier” event occurred.

Now, we will consider the probability of an event “earlier” in the tree, given that a “later” event occurs.

**Example 1:** A box contains 4 white marbles and 3 red marbles. Two marbles are drawn in succession without replacement. What is the probability of drawing a red marble on the first draw given that a red marble is drawn on the second draw?

We’ve just used something a theorem known as Bayes’ Theorem. You don’t need to memorize it, as it is just the conditional probability formula $P(A \mid B) = \frac{P(A \cap B)}{P(B)}$.

**Bayes’ Formula:**

$$P(A \mid B) = \frac{\text{product of probabilities on path to B through A}}{\text{sum of all branch products leading to B}}$$
Example 2: The picture tubes for the Pulsar 19-inch television sets are manufactured in three locations and then shipped to the main plant for final assembly. Plants A, B, and C supply 50%, 30%, and 20%, respectively, of the picture tubes. The quality-control department of the company has determined that 1% of the picture tubes produced by plant A are defective, whereas 2% of the tubes produced by plants B and C are defective. If a Pulsar 19-inch color television is selected at random and the picture tube is found to be defective, what is the probability that the picture tube was manufactured in plant C?