8.1: Sample Spaces, Events, and Probability

An experiment is an activity with observable results. An experiment that does not always give the same result, even under the same conditions, is called a random experiment. Repetitions of an experiment are called trials.

Examples: rolling dice, drawing cards, counting the number of defective radios in a shipment, etc.

In probability theory, we’ll usually use the word experiment to mean a random experiment.

For a given experiment, we can make a list of outcomes of the experiment, called simple events, such that in each trial, one and only one of the simple events will occur. The set of all such simple events is called the sample space.

Any subset of the sample space is called an event. If such a subset contains more than one element of the sample space, we call the subset a compound event. If an event \( E = \emptyset \), we call the event \( E \) an impossible event. If \( S \) is the sample space and \( E = S \), then \( E \) is a certain event.

Example 1: We roll a single six-sided die.

Example 2: We spin a roulette wheel (a wheel that has the integers 0 to 36 on it).

Example 3: The manager of a local cinema records the number of patrons attending a particular movie. The theater has a seating capacity of 500.

a. What is an appropriate sample space for this experiment?

b. Describe the event \( E \) that fewer than 50 people attend the movie?

c. Describe the event \( F \) that the theater is more than half full at the movie?
Example 4: An experiment consists of studying the composition of a three-child family.
   a. Describe an appropriate sample space for this experiment if we are interested in the
genders of the children in the order of their births.
   b. Describe the event $E$ that there are two girls and a boy in the family.
   c. Describe the event $F$ that the oldest child is a girl.
   d. Describe the event $G$ that the oldest child is a girl and the youngest is a boy.
   e. Describe an appropriate sample space if we are only interested in the number of girls
and not the order.

Example 5: Roll a pair of dice. What is the most useful sample space? Describe the
following events:
   a. The sum is 6.
   b. The sum is 2.
   c. The sum is more than 9.
8.1.3

**Probability:**

In many sample spaces, all the outcomes, or simple events, are equally likely to occur. In these cases, we use the *equally likely assumption*. If several choices are possible for an experiment’s sample space, it is often best to choose one in which all outcomes are equally likely.

<table>
<thead>
<tr>
<th>Equally likely assumption:</th>
</tr>
</thead>
<tbody>
<tr>
<td>If all events in a sample space are equally likely to occur, the probability of each is ( \frac{1}{n} ), where ( n ) is the number of simple events in the sample space.</td>
</tr>
</tbody>
</table>

The equally likely assumption results in a basic principle of probability.

<table>
<thead>
<tr>
<th>Basic Probability Principle</th>
</tr>
</thead>
<tbody>
<tr>
<td>Let ( S ) be a sample space of equally likely outcomes, and let the event ( E ) be a subset of ( S ). Then the probability that event ( E ) occurs is</td>
</tr>
<tr>
<td>( P(E) = \frac{n(E)}{n(S)} ).</td>
</tr>
</tbody>
</table>

**Example 6:** Flip two fair coins. What is the probability of getting
a. Two tails?
b. At least one head?
c. Exactly one head?
d. Two heads or two tails?
e. Three tails?
Example 7: Draw a single card from a standard card deck. What is the probability that it is
a. A spade?
b. A queen?
c. Black?
d. Higher than a 7? (assume aces high.)

Example 8: Roll a single die. What is the probability of rolling
a) a 5?
b) a prime number?
c) a multiple of 3?
d) a number larger than 10?
e) a number smaller than 10?
Example 9: Roll two dice. What is the probability that the sum is 7? That the sum is less than 6?

Example 10: Jennifer’s electricity went out last night. Not having a flashlight, she had to choose a T-shirt at random in the dark. She owns 10 white T-shirts, 5 gray ones, 6 black ones and 2 red ones. What is the probability that she chose a gray T-shirt?

Example 11: Suppose that Joe, Steve, Suzy, and Lisa work for the same company. The company wants to send two representatives to a particular conference and needs two to stay home and take care of the customers. All of them want to attend the conference, so they decide to put their names in a hat and draw two at random. What is the probability that Suzy and Lisa are selected?
**Example 12:** The tuition categories for North Harris College students in the spring semester of 2007 are given in the table below.

<table>
<thead>
<tr>
<th>Category</th>
<th>Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>In-District</td>
<td>9,794</td>
</tr>
<tr>
<td>Out-of-District</td>
<td>428</td>
</tr>
<tr>
<td>Out-of-State</td>
<td>104</td>
</tr>
<tr>
<td>International</td>
<td>250</td>
</tr>
<tr>
<td>Other</td>
<td>9</td>
</tr>
</tbody>
</table>

a) What is the probability that a randomly selected student lives in-district?

b) What is the probability that a randomly selected student is an international student?

**Example 13:** The age distribution of Spring 2007 North Harris College students in the spring semester of 2007 is given in the table below.

<table>
<thead>
<tr>
<th>Age of Students</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Under 20</td>
<td>33.4%</td>
</tr>
<tr>
<td>20-24</td>
<td>35.6%</td>
</tr>
<tr>
<td>25-29</td>
<td>11.3%</td>
</tr>
<tr>
<td>30-39</td>
<td>11.5%</td>
</tr>
<tr>
<td>40-49</td>
<td>6.3%</td>
</tr>
<tr>
<td>50 and Over</td>
<td>2.0%</td>
</tr>
<tr>
<td>Total</td>
<td>100%</td>
</tr>
</tbody>
</table>

What is the probability that a randomly selected student is

a) under 20 years old?

b) Over 40?

c) Over 60?
Probability assignments:

Suppose \( S = \{e_1, e_2, \ldots, e_n\} \) is a sample space. It contains \( n \) simple events, or outcomes. To each outcome, we can assign a number \( P(e_i) \), called the probability of the event \( e_i \).

Rules for probabilities:

- The probability of a simple event is a number between 0 and 1, inclusive. In other words, \( 0 \leq P(e_i) \leq 1 \).
- The probabilities of all the simple events in the sample space add up to 1. In other words, \( P(e_1) + P(e_2) + \ldots + P(e_n) = 1 \).

Any probability assignment that meets these conditions is called an acceptable probability assignment.

A probability assignment that reflects the actual or expected percentage of times a simple event occurs is called reasonable. In other words, it is reasonable if it makes sense based on the real world.

Example 14: Roll a single die.

Probability of an event \( E \):

Given a sample space \( S \) and an acceptable probability assignment, then for any event \( E \) (\( E \subset S \)), the following rules apply:

- If \( E = \emptyset \), then \( P(E) = 0 \).
- If \( E \) is a simple event, then \( P(E) \) is given by the original probability assignment.
- If \( E \) is a compound event, then \( P(E) \) is the sum of the probabilities of all the simple events in \( E \).
- If \( E = S \), then \( P(E) = P(S) = 1 \).

Steps for finding \( P(E) \):

1. Set up an appropriate sample space.
2. Assign acceptable probabilities to each simple event.
3. \( P(E) \) is the sum of the probabilities of all the simple events in \( E \).
There are two basic methods of assigning probabilities.

- **Theoretical**: no experiments are conducted—use assumptions and reasoning.
- **Empirical**: use results of actual experiments. (This is also called *relative frequency probability*.)

**Empirical (relative frequency) probability approximation**:

\[
P(E) \approx \frac{\text{frequency of occurrence of } E}{\text{total number of trials}} = \frac{f(E)}{n}
\]

(The larger \( n \) is, the better the approximation.)

**Example 15**: Suppose that we survey 134 students at Lone Star College-North Harris and find that 47 of them own an iPod. Use this information to estimate the probability that a randomly selected LSC-NH student owns an iPod. What is the probability that the student does not own an iPod?